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Symmetry and the order of events in time: the thermodynamics of blackbodies composed of positive or negative mass

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Abstract: I have previously proposed a discrete symmetry called CPM (charge-parity-mass) symmetry that defines matter as having positive mass and antimatter as having negative mass—both being involved in processes that proceed forward in time. I have generalized the second law of thermodynamics to describe and predict the order of events in time that take place in reversible thermodynamic systems composed of positive or negative mass. Here I extend the CPM symmetry to irreversible thermo-optical radiation systems characterized by the Stefan–Boltzmann law, Planck's blackbody radiation law, and Einstein's law of specific heat. The laws of radiation analyzed in terms of CPM symmetry may provide a fresh way of understanding old topics such as the photon and entropy and new topics such as the nonluminous substances in the universe known as dark matter.

Key words: Antimatter, blackbody radiation, dark matter, entropy, negative mass, photon, thermodynamics

1. Introduction

In material systems composed of positive mass, entropy flows from hotter bodies to colder bodies as described by the second law of thermodynamics [1]. Coincident with the flow of entropy, and as long as there are no phase changes, colder bodies in communication with hotter bodies become hotter and the hotter bodies become cooler. The rate of equilibration depends on the specific heats of the bodies. The flow of entropy results from radiative, convective, and/or conductive processes. Entropy flow can also occur in the absence of a temperature change. For example, when a solid substance composed of positive mass is in thermal communication with a hotter environment, the solid may melt or sublime as entropy passes to the solid made of positive mass. Likewise, entropy can flow from a hot body made of positive mass to a cooler environment. The flow of entropy to or from positive mass bodies can be modeled by the flow of photons.

Isaac Newton [2] defined a volume of matter to have a positive mass and time to be absolute. Using charge-parity-mass (CPM) symmetry, I have shown that matter in the modern sense can be considered to have a positive inertial and gravitational mass while antimatter can be considered to have a negative inertial and gravitational mass, consistent with the equivalence principle [3]. Similar considerations have been made by Bo Lehnert [4], although he considered antimatter to have a negative gravitational mass and a positive inertial mass. CPM symmetry is based on the assumption that time is absolute, Newtonian, unidirectional, and irreversible for both matter and antimatter [3]. CPM symmetry is only one possible symmetry that can be used to describe and predict the relationship between matter and antimatter. Indeed, charge-parity-time (CPT)

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symmetry, which is based on the assumption that time is relative, is the most accepted symmetry. There are two interpretations of CPT symmetry. One, championed by Feynman [5], considers time to be bidirectional and flowing in one direction for matter and in the opposite direction for antimatter, both of which are considered to have positive mass. Consequently, antimatter is interpreted to behave as if it were matter going "backwards in time." The other interpretation, championed by Recami and coworkers [6–8], interprets matter as having positive rest mass traveling forward in time and antimatter as having negative rest mass traveling backwards in time. However, due to their principle of reinterpretation (RIP), which asserts that physical signals are transported only by objects that appear with positive energy travelling forward in time, two negatives make a positive and antimatter is experienced as positive mass traveling forward in time. If CPT symmetry based on ordinary physics and special relativity or extended relativity be the true symmetry, the following analysis would be moot. However, if CPM be the true symmetry, the following analysis would have descriptive and predictive power. While both symmetries are based on untested assumptions, the veracity of the symmetries is subject to observational tests. As will be discussed below, CPM symmetry predicts that dark matter is equivalent to antimatter.

I have derived a generalized second law of thermodynamics that is consistent with CPM symmetry [9]. According to this law, with substances made of negative mass, entropy does not pass from hotter to colder substances backwards in time but passes from colder to hotter bodies forward in time. Consequently, when a test substance of negative mass is in thermal communication with a hotter environment, entropy will flow from the cold negative mass body to the hotter environment and the negative mass body will get cooler [9]. Entropy flow from a cold body made of negative mass can also occur in the absence of a temperature change. For example, when a gaseous negative mass substance is in thermal communication with a hotter environment, it can also undergo a phase change and become a liquid or solid. Likewise, entropy can flow from a colder environment into a hot body composed of negative mass and the negative mass body will get hotter. The flow of entropy to or from a body composed of negative mass can also be modeled by the flow of photons.

The order of events in time for substances composed of either positive or negative mass is correlated with the direction of flow of entropy. In a Carnot engine made of positive mass, exposure to radiation from a hot body composed of positive mass results in expansion [1]. This isothermal expansion takes place at high temperature while the isothermal compression takes place at low temperature; consequently, entropy flows though the Carnot engine made of positive mass from the hot reservoir to the cold reservoir.

In a Carnot engine made of negative mass, exposure to radiation from a cold body composed of positive mass results in expansion [9]. This isothermal expansion takes place at low temperature while isothermal compression takes place at high temperature; consequently, entropy flows through the Carnot engine made of negative mass from the cold reservoir to the hot reservoir. I have previously used CPM symmetry to describe and predict the flow of entropy in reversible thermodynamic systems [1,9]. Here I explore CPM symmetry in irreversible systems such as blackbodies.

2. Stefan–Boltzmann law for bodies made of positive or negative mass

A scientific understanding of age-old questions concerning the age of the Earth and the energy it gets from the Sun's radiation were sought in measuring the rates of heating and cooling of various objects [10–18]. The rate of cooling is roughly proportional to the temperature of a body as noted by Isaac Newton [19–24]. However, cooling is a complicated process that can occur through various mechanisms, including conduction and convection, as well as through the radiation of heat. Dulong and Petit [25], studying the rate of cooling of a thermometer, found that the equation that describes cooling is not simply a function of temperature, but involves several influences such as the nature and concentration of the gas that surrounds the hot body.

The transfer of heat by convection was studied by Count Rumford (Benjamin Thompson) [26]. The transfer of heat by conduction was analyzed and modeled by Joseph Fourier [27]. The transfer of heat by radiation was studied by John Draper [28–30], John Tyndall [31–36], and John Ericsson [37], who characterized the radiation of heat by measuring the quantity of radiation emitted by platinum or iron held at different temperatures. Josef Stefan noticed that the amount of radiation emitted by a hot body could be described as proportional to the fourth power of the temperature [38–52]. Ludwig Boltzmann [53,54] reified Stefan's hypothesis by treating electromagnetic radiation as an analog to a gas surrounded by walls using already established relationships between energy density and pressure [55–58]. Experiments by Lummer and Pringsheim [59] on the radiation produced in the cavity of a blackbody held at constant temperatures ranging from 373.1 K to 1561 K confirmed the theoretical formulae of Stefan and Boltzmann. Here I confirm and extend Boltzmann's thermodynamic derivation in the context of CPM symmetry.

Let's start with the fundamental equation of thermodynamics for positive and negative mass [9]:

$$\aleph T dS = dU + \aleph p dV, \tag{1}$$

where $\aleph$ is the atomic mass standard coefficient equal to +1 for a system composed of positive mass or −1 for a system composed of negative mass [9]; T is the absolute temperature; S is entropy; U is the internal energy, which is a function of temperature and is greater than zero for positive mass and less than zero for negative mass; p is pressure; and V is volume. Let u be the internal energy density (in J/m^3 = N/m^2 = Pa), which is dimensionally the same as pressure, and write $U = Vu$ and $\aleph p = \frac{u}{3}$:

$$\aleph T dS = d(Vu) + \frac{u}{3} dV. \tag{2}$$

Note that $d(Vu) = Vdu + udV$ by the chain rule. Clear T from the L.H.S and then multiply the first term on the R.H.S. by $\frac{dT}{dT} = 1$ to get:

$$\aleph dS = \frac{V}{T}\frac{du}{dT} dT + \frac{4u}{3T} dV. \tag{3}$$

Now $\aleph dS$ is a complete differential so we can also write:

$$\aleph dS = \aleph \frac{\partial S}{\partial T} dT + \aleph \frac{\partial S}{\partial V} dV. \tag{4}$$

Comparing coefficients of dT and dV from Eqs. (3) and (4), we see:

$$\aleph \frac{\partial S}{\partial T} = \frac{V}{T}\frac{du}{dT} \tag{5}$$

and

$$\aleph \frac{\partial S}{\partial V} = \frac{4u}{3T}. \tag{6}$$

Differentiating both sides of Eq. (5) with respect to V, we get:

$$\frac{\partial}{\partial V}\aleph \frac{\partial S}{\partial T} = \aleph \frac{\partial^2 S}{\partial V \partial T} = \frac{\partial}{\partial V}\frac{V}{T}\frac{du}{dT} = \frac{1}{T}\frac{du}{dT}. \tag{7}$$

Differentiating both sides of Eq. (6) with respect to T, we get:

$$\frac{\partial}{\partial T}\aleph\frac{\partial S}{\partial V}=\aleph\frac{\partial^2 S}{\partial V\partial T}=\frac{4}{3}\frac{d\frac{u}{T}}{dT}. \tag{8}$$

Eqs. (7) and (8) both contain $\aleph\frac{\partial^2 S}{\partial V\partial T}$. Consequently,

$$\aleph\frac{\partial^2 S}{\partial V\partial T}=\frac{4}{3}\frac{d\frac{u}{T}}{dT}=\frac{1}{T}\frac{du}{dT}. \tag{9}$$

Expanding the derivatives in the middle term, we get:

$$\frac{4}{3}\left[\frac{du}{TdT}-\frac{udT}{T^2dT}\right]=\frac{1}{T}\frac{du}{dT}. \tag{10}$$

Simplifying, we get:

$$\frac{4\,du}{3\,TdT}-\frac{4udT}{3T^2dT}=\frac{1}{T}\frac{du}{dT}. \tag{11}$$

Subtracting and rearranging, we get:

$$\frac{du}{3T\,dT}=\frac{4u\,dT}{3T^2dT}. \tag{12}$$

Cancelling like terms, we get:

$$\frac{du}{dT}=\frac{4u}{T}. \tag{13}$$

Separating the variables, we get:

$$\frac{du}{u}=4\frac{dT}{T}. \tag{14}$$

Then we integrate Eq. (14):

$$\int\frac{1}{u}du=4\int\frac{1}{T}dT. \tag{15}$$

After solving the indefinite integral, we get:

$$\ln(u)=4\ln(T)\pm\ln(a), \tag{16}$$

where a is the constant of integration that is positive. We can solve Eq. (16) for the temperature and internal energy density differences between the hot body (hb) and the environment (e) after substituting $\aleph$ for the $\pm$ sign where $\aleph=+1$ for systems composed of positive mass and -1 for systems composed of negative mass:

$$u_{hb}-u_e=\aleph a\left[T_{hb}^4-T_e^4\right]. \tag{17}$$

When the temperature of any hot body is much greater than the temperature of the environment, Eq. (17) reduces to

$$u=\aleph aT^4 \tag{18}$$

under CPM symmetry where $\aleph a>0$ for substances composed of positive mass and $\aleph a<0$ for substances composed of negative mass. Define R (in J m^{-2} s^{-1}) as the flux of radiation or power per unit area emitted from a blackbody where $R > 0$ when there is a net emission from a blackbody and $R < 0$ when there is a net absorption by a blackbody. The radiant flux (R) is related to the internal energy density (u) by the following equation:

$$\Re = \frac{c}{4}u, \tag{19}$$

where c is the speed of light.

The total flux of photons from a blackbody radiator made of positive or negative mass is given by the following equation obtained by combining Eqs. (18) and (19):

$$\Re = \frac{ac}{4}\aleph T^4 = \sigma \aleph T^4, \tag{20}$$

where σ denotes the Stefan–Boltzmann constant ($\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.6693 \times 10^{-8}$ J s m^{-2} K^{-4}) and is equal to $\frac{ac}{4}$. The Stefan–Boltzmann constant is always positive, although $\aleph\sigma > 0$ for positive masses and $\aleph\sigma < 0$ for negative masses. We know that a blackbody composed of matter in outer space with a temperature greater than the temperature of the cosmic microwave background radiation will radiate photons and cool with time. Since CPM symmetry asserts that time is absolute and unidirectional [60], the generalized Stefan–Boltzmann law given in Eq. (20) predicts that a blackbody composed of negative mass in outer space with a temperature greater than the temperature of the cosmic microwave background radiation will absorb photons and get hotter over time instead of radiating photons backwards in time as CPT would hold. Moreover, as the temperature decreases in a blackbody made of positive mass, it emits less radiation. When the temperature increases in a blackbody made of negative mass, it absorbs more radiation.

3. Planck's radiation law for blackbodies made of positive and negative mass

Before examining Planck's radiation law in the context of CPM symmetry, I will briefly trace the origin and development of Planck's radiation law.

The promotion of the useful arts through science has contributed to our understanding of blackbodies. Josiah Wedgwood [61,62] invented the clay pyrometer to measure the temperature of the kilns he used to fire his pottery in order to reproducibly obtain the optimal temperature for firing pottery. His son Thomas Wedgwood [63,64] noticed that all the objects in the kiln became incandescent or luminous in the dark at the same temperature. John Draper [11,30] also noticed that when he heated up various substances, they became incandescent and that the color of the incandescence changed as the temperature increased. Pierre Prévost [65] realized that, by definition, at thermal equilibrium, a substance receives as much heat as it radiates. John Leslie [66,67], William Ritchie [68], and Balfour Stewart [69–71] determined the fact, now commonplace, that dark surfaces emit more heat radiation than bright surfaces and that the ability of the surface of most materials to absorb heat is proportional to its ability to radiate heat.

Gustav Kirchhoff [72–75] created a general theorem of radiation that stated that at thermal equilibrium, the radiation from a blackbody is independent of the nature of the material that forms the blackbody. Moreover, if a small hole is made in a wall of the blackbody, the amount of radiation of any wavelength emitted through the hole will depend only on the temperature of the blackbody. Kirchhoff declared that a universal function must exist that describes the spectral emission of a blackbody solely in terms of its temperature and independent of the material that makes up the blackbody.

Wilhelm Wien [76,77] used thermodynamic arguments to try to discover the universal function proposed by Kirchhoff to relate the peak wavelength of cavity radiation to the temperature of the cavity. Wien contended that if the radiation were in a mirrored box, the incident and reflected radiation would exert a pressure on the walls. Thus, if one did work to shrink the volume of the box against the pressure of the radiation, two things would happen as the volume of the box decreased. Firstly, the wavelengths would shorten due to the Doppler effect, and secondly, the energy density would increase, which would result in an increase in the temperature of the radiation. Thus, by imagining the effect of doing mechanical work on the radiation field, he could determine the relationship between the peak wavelength and the temperature.

Starting with the fact that the radiation emitted or absorbed by a blackbody in the spectral range from λ to $\lambda + d\lambda$ is a function of temperature, we can rewrite the generalized Stefan–Boltzmann law in terms of the radiation in a given spectral range emitted by a blackbody at rest made of positive mass and thus having positive total energy or a blackbody at rest made of negative mass [7] and thus having negative total energy:

$$\Re = \int_0^\infty \Re(\lambda) d\lambda = \sigma \aleph T^4. \tag{21}$$

The introduction of $\aleph$ into the equation emphasizes under CPM symmetry that the spectral distribution of radiation is not solely a function of temperature as stated by Kirchhoff, but is also a function of the sign of the mass of the blackbody.

Next we follow Wien in assuming that there are two functions (f and F) that relate $R(\lambda)d\lambda$ to temperature (Eq. (22)) or to wavelength (Eq. (23)):

$$\Re\left(\lambda\right) d\lambda \; = \; T^5 f(\lambda T), \tag{22}$$

$$\Re\left(\lambda\right) d\lambda \; = \lambda^{-5} F(\lambda T). \tag{23}$$

The two functions are related in the following way:

$$F(\lambda T) = \left(\lambda T\right)^5 f(\lambda T). \tag{24}$$

Given the Stefan–Boltzmann law that that the integral of the flux of radiation is proportional to the fourth power of the temperature, the radiation at any given wavelength must be equal to the product of a function to be determined and the fifth power of the temperature. Wien set up the relationship between the universal function that applied to all black bodies composed of positive mass and temperature:

$$f(\lambda T) = \frac{\Re(\lambda) d\lambda}{T^5}. \tag{25}$$

Given the similarity in the shapes of the curves describing blackbody radiation and Maxwell's distribution of the velocities of molecules, Wien guessed an exponential form of the universal function by assuming that 1) the radiation was emitted by atomic oscillators, 2) the frequency of the emitted radiation was proportional to the kinetic energy of the oscillators, and 3) the intensity at a given frequency of radiation was proportional to the number of oscillators with a kinetic energy proportional to the observed frequency. He proposed the following equation:

$$\Re\left(\lambda\right) d\lambda = \frac{c}{4} u\left(\lambda\right) d\lambda = \frac{c}{4}\frac{C_1}{\lambda} e^{\frac{-C_2}{\lambda T}}\, d\lambda. \tag{26}$$

By 1900, it was clear that, with the proper choice of the constants C_1 and C_2, Wien's law fit the experimental data obtained by Lummer and Pringsheim [78,79] and Rubens and Kurlbaum [80] closely at low values of λT but failed at higher values, where it gave values that were too low. On the other hand, the formula presented by Lord Rayleigh [81] that was based on the principle of equipartition accounted for the experimental values at higher values of λT but failed at lower values. Taking a thermodynamic approach, Max Planck [82–87] continued the search for the universal function that would more accurately describe blackbody radiation. He too treated the radiation in the cavity of a blackbody as an isolated thermodynamic system at constant volume (V) and temperature (T) composed of radiation that could exert a pressure (p) against the walls. At equilibrium, absorption is equal to emission and the energy density in the cavity is equal to the energy density of the walls.

Planck considered the fundamental units of the walls of the blackbody radiator to consist of harmonic oscillators that absorbed and emitted radiation and whose average internal energy (U_{ave}) varied with temperature. Given that the thermodynamic temperature of an oscillator could be defined by the derivative of its average internal energy with respect to its average entropy, Planck searched for a way of characterizing the average entropy (S_{ave}) of an oscillator in the wall of the blackbody radiator. In his struggle, Planck incorporated Boltzmann's statistical reasoning that considered entropy (S) to be related logarithmically to the thermodynamic probability ($S = k \ln W$, where W comes from Wahrscheinlichkeit) of a given configuration. The constant (k) that relates entropy to thermodynamic probability was named Boltzmann's constant by Planck. By assuming that each microstate is as likely to occur as any other microstate, Boltzmann had determined the thermodynamic probability (W) by counting the number of physically meaningful microstates. In applying statistical mechanics to blackbody radiation, Planck used Boltzmann's rationale but not his combinational procedure, which, based upon the assumption of infinitely divisible and continuously varying quantities, would have resulted in Wien's radiation law.

In his search for a definition of entropy that would yield the universal function that accurately described the experimental data, Planck calculated the thermodynamic probability of various distributions of electromagnetic radiation and deduced the average entropy of the oscillators with a novel postulated definition of thermodynamic probability. Planck warily introduced the postulate that the energies of the oscillators in the wall were not distributed continuously, but could only exist in discrete energy levels, known as quanta.

By assuming that each oscillator could only take the values $0U_j$, $1U_j$, $2U_j$, $3U_j$, $4U_j$, etc., Planck discovered the distribution for P units of energy among N oscillators in the walls of the blackbody that would give the average energy of the oscillators that would fit in an equation that matched the experimental data. Planck found that the thermodynamic probability (W) of N oscillators with P units of energy was:

$$W = \frac{(P+N-1)!}{P!\,(N-1)!}, \tag{27}$$

which is the number of all the physically meaningful macrostates.

At equilibrium, the energy density of Planck's oscillators in the wall of the cavity equals the energy density of the radiation in the cavity itself [88]. By combining his model of the light quantum with Bohr's model of light absorption and emission by atoms, Einstein [89–94] realized that "Planck's theory makes implicit use of the aforementioned hypothesis of light quanta," and he turned his attention from Planck's quantized oscillators in the wall of the cavity that could absorb and emit radiation to the quantized radiation in the cavity itself.

Building on Einstein's realization but not his model of the light quantum, I will assume that the cavity is composed of photons with extension in space [95–97] and that the photons of a given frequency are indistinguishable. The total number of arrangements of photons is equal to the number of ways P indistinguishible photons of a given frequency can be distributed among N indistinguishable spaces in the cavity. However, since the photons of a given frequency and the volume of space that they might take up are indistinguishable, all permutations that differ only in the positioning of the photons in space are physically the same. The quantization of space is justified by the idea that even the smallest photon in the cavity has extension and takes up space and does not imply that the space itself is quantized. Moreover, any attempt at individualization or numbering each separate photon or each space in the cavity is meaningless since the given state is completely and uniquely described by specifying how many photons are in the cavity [88]. Therefore, we have to divide the total number of possible permutations, which is $(N+P-1)!$ by $N!(P-1)!$ in order to eliminate physically meaningless distinctions. Consequently, the thermodynamic probability, which is equal to the total number of physically meaningful arrangements, is $\frac{(P+N-1)!}{N!(P-1)!}$. While this number represents the total number of physically meaningful arrangements and not the most probable arrangement, when P and N are large, the total number of physically meaningful arrangements is approximately equal to the one or two most probable arrangements.

The number of physically meaningful arrangements of P photons among N spaces in the cavity is related to the entropy (S) of the photon gas and is given by the following equation:

$$\aleph S = \aleph k \ln[W] = \aleph k \ln\left[\frac{(N+P-1)!}{N!\,(P-1)!}\right]. \tag{28}$$

Note that $\aleph$, which represents the sign of the mass, cancels from all terms and thus the relationship between entropy and thermodynamic probability is independent of the sign of the mass. When $N >> 1$ and $P >> 1$, Eq. (28) reduces to:

$$\aleph S = \aleph k \ln\left[\frac{(N+P)!}{N!\,P!}\right]. \tag{29}$$

Using Stirling's approximation for evaluating factorials of large numbers, where $\ln N! \approx N\ln N - N$, Eq. (29) becomes:

$$\aleph S = \aleph k\left[(N+P)\ln(N+P) - N\ln N - P\ln P\right]. \tag{30}$$

We can solve for the average entropy (S_{ave}) of a space in the cavity surrounded by positive or negative mass by dividing by the number of spaces (N) in the cavity:

$$\aleph S_{ave} = \frac{\aleph S}{N} = \aleph k\left[(1+\frac{P}{N})\ln(1+\frac{P}{N}) - \ln 1 - \frac{P}{N}\ln\frac{P}{N}\right]. \tag{31}$$

The internal energy (U) in the cavity is given by the product of the number (P) of photons in the cavity times the energy of those photons $(\aleph h\nu)$. The sign of the internal energy (U) can be either positive or negative depending on the sign of the mass of the walls, and while $h\nu$ is always positive, the sign of the energy of the photon $(\aleph h\nu)$ depends on the sign of the mass of the cavity walls. The internal energy is also given by the product of the number of spaces (N) in the cavity times the average internal energy of the spaces (U_{ave}). Letting $U = P\aleph h\nu = NU_{ave}$ and $\frac{P}{N} = \frac{U_{ave}}{\aleph h\nu}$, Eq. (31) becomes:

$$\aleph S_{ave} = \aleph k\left[(1+\frac{U_{ave}}{\aleph h\nu})\ln(1+\frac{U_{ave}}{\aleph h\nu}) - \frac{U_{ave}}{\aleph h\nu}\ln\frac{U_{ave}}{\aleph h\nu}\right]. \tag{32}$$

We can find the average internal energy of a space in the cavity from the average entropy of the space using the definition $T \equiv \frac{\partial U}{\aleph \partial S}$, where T is always positive. Since $\frac{1}{T} = \frac{\aleph \partial S_{ave}}{\partial U_{ave}}$ for positive and negative mass bodies at constant volume [9], Eq. (32) becomes:

$$\frac{1}{T} = \frac{\partial}{\partial U_{ave}} \aleph k \left[(1 + \frac{U_{ave}}{\aleph h\nu}) \ln (1 + \frac{U_{ave}}{\aleph h\nu}) - \frac{U_{ave}}{\aleph h\nu} \ln \frac{U_{ave}}{\aleph h\nu} \right] \tag{33}$$

Solving for the derivative in Eq. (33), we get:

$$\frac{1}{T} = \aleph k \frac{\ln \left[\frac{\aleph h\nu + U_{ave}}{U_{ave}} \right]}{\aleph h\nu}. \tag{34}$$

Cancelling like terms and simplifying we get:

$$\frac{1}{T} = \frac{\aleph k}{\aleph h\nu} \ln \left[1 + \frac{\aleph h\nu}{U_{ave}} \right] = \frac{k}{h\nu} \ln \left[1 + \frac{\aleph h\nu}{U_{ave}} \right]. \tag{35}$$

After rearranging, we get:

$$\ln \left[1 + \frac{\aleph h\nu}{U_{ave}} \right] = \frac{h\nu}{kT}. \tag{36}$$

After exponentiating, we get:

$$1 + \frac{\aleph h\nu}{U_{ave}} = e^{\frac{h\nu}{kT}}. \tag{37}$$

After rearranging, we get:

$$\frac{\aleph h\nu}{U_{ave}} = e^{\frac{h\nu}{kT}} - 1. \tag{38}$$

After solving for the average internal energy of a space in the cavity, we get:

$$U_{ave} = \frac{\aleph h\nu}{e^{\frac{h\nu}{kT}} - 1}. \tag{39}$$

The internal energy of a positive mass ($\aleph > 0$) will become more positive when a photon is absorbed, and internal energy of a negative mass ($\aleph < 0$) will become more negative when a photon is absorbed. The internal energy of a positive mass ($\aleph > 0$) will become less positive when a photon is emitted, and the internal energy of a negative mass ($\aleph < 0$) will become less negative when a photon is emitted.

Eq. (39) gives the average internal energy in the cavity of a given frequency. The sign of the average internal energy depends on the sign of the mass of the blackbody. The number of photons in the cavity does not depend on the sign of the mass of the blackbody. According to the binary photon model [95–97], $\frac{8\pi V \nu^2}{c^3} d\nu$ photons of frequencies $\nu + d\nu$ can fit in a cavity of volume (V). The actual number of photons of a given frequency in the cavity at a given temperature is obtained by multiplying the volume-dependent and mass-independent term by the temperature- and mass-dependent term. The internal energy (U) of radiation in the cavity is:

$$U = \frac{8\pi V \nu^2}{c^3} \frac{\aleph h\nu}{e^{\frac{h\nu}{kT}} - 1} d\nu, \tag{40}$$

and the internal energy density for a given frequency is:

$$u = \frac{U}{V} = \frac{8\pi\nu^2}{c^3}\frac{\aleph h\nu}{e^{\frac{h\nu}{kT}}-1}\,d\nu. \tag{41}$$

Substituting Eq. (41) into Eq. (19), we get radiant flux (R) for a blackbody radiator composed of positive or negative mass:

$$\Re = \frac{2\nu^2}{c^2}\frac{\aleph h\nu}{e^{\frac{h\nu}{kT}}-1}\,d\nu. \tag{42}$$

Thus, for a given temperature, the direction of the radiant flux is either outward or inward, being opposite for a blackbody made of positive mass ($\aleph > 0$) and one made of negative mass ($\aleph < 0$). By CPM symmetry, for blackbodies that are hotter than the environment, the positive mass blackbody will act as an emitter and the negative mass blackbody will act as an absorber relative to the environment.

We further see from Eq. (42) that the spectral distribution of a blackbody composed of positive mass or negative mass is the same. For a blackbody composed of positive mass in outer space with a temperature greater than the temperature of the cosmic microwave background radiation, the spectrum will be an emission spectrum and the blackbody will radiate photons and cool. By contrast, for a blackbody composed of negative mass in outer space with a temperature greater than the temperature of the cosmic microwave background radiation, the spectrum will represent an absorption spectrum and the blackbody will absorb photons and heat up. The proposed absorption and emission properties of antimatter considered to have a negative mass could be tested with antihydrogen atoms at different temperatures. CPM symmetry predicts that when antimatter is placed in a spectrophotometer, the absorption of light by the antiatoms should increase when their temperature is increased relative to their environment and emission of light by the antiatoms should decrease when their temperature is increased relative to the environment. Since antimatter, considered under CPM symmetry to have a negative mass, would absorb electromagnetic radiation from a colder environment [9] and interact gravitationally with matter with a positive mass [98], it possible that antimatter might account for at least some of the nonluminous "dark matter" in the universe.

4. An extension of Einstein's treatment of specific heat for positive mass extended to negative mass

Joseph Black [99,100] characterized the capacity of a substance to hold heat and Clausius [101] suggested that the heat can be stored, not only in the motions that result in translation, but also in vibrational and rotational motions. In this manner, the specific heats of substances were related to their internal motions and their internal motions could be predicted from the specific heats. Eq. (1), the fundamental equation of thermodynamics under CPM symmetry [9], can be written in terms of the specific heats at constant volume (C_v) and constant pressure (C_p):

$$\aleph T dS = C_v dT + ((C_p - C_v)\frac{T}{V})dV. \tag{43}$$

According to CPM symmetry, the specific heats are positive for matter with a positive mass and negative for antimatter with a negative mass [9]. An increase in the temperature of a positive mass occurs when it is in communication with a hotter body and an increase in the temperature of a negative mass occurs when it is in communication with a colder body. On the other hand, coincident with an increase in temperature, the entropy of both positive and negative mass substances increases. Is there a universal carrier of entropy that causes the temperature of all substances to increase, and, if so, what is its identity? In order to answer this question, we will first look at the quantum theory of specific heat, which puts limitations on the motion of oscillators at low temperature.

Upon measuring the specific heats at constant volume of solids such as diamond, Heinrich Weber [102,103] realized that the specific heat at constant volume was not a constant as implied by the results of Petit and Dulong [104], but varied with the temperature of the substance. In an attempt to illuminate the relationship between specific heat and temperature, Albert Einstein [105–107] constructed a quantum theory of specific heat. Although Einstein's conception of quanta differed from that of Planck, Einstein was one of the few physicists who, along with Johannes Stark [108], took Planck's quantum theory seriously at the time [87,109–113]. Most physicists considered Planck's radiation law to be a form of curve fitting and expected a return to classical ideas that would be described by differential equations based on the assumption of continuously variable quantities whose limits approached zero. Einstein realized that if Planck's blackbody radiation law applied to specific heats, then the temperature-dependent radiation of heat would have a peak wavelength characterized by the same universal function. Einstein's quantum theory of specific heat described the temperature-dependence of the specific heat of solids. Here I generalize the quantum theory of specific heat to substances with negative mass.

Einstein made the simplifying assumption that in all solids, the oscillators vibrate at the same frequency and the internal energy per mole (U_{mol}) is given by Planck's blackbody radiation formula, here expressed under CPM symmetry:

$$U_{mol} = \frac{N_A \aleph h\nu}{e^{\frac{h\nu}{kT}} - 1}, \tag{44}$$

where N_A is Avogadro's number.

The above equation was derived by assuming independent linear harmonic oscillators. Einstein assumed that each oscillator in a solid could vibrate in three independent directions and thus act like three independent linear harmonic oscillators. Consequently he multiplied the right hand side of Eq. (44) by 3 to yield the number of linear harmonic oscillators per mole:

$$U_{mol} = \frac{3N_A \aleph h\nu}{e^{\frac{h\nu}{kT}} - 1}. \tag{45}$$

By differentiating Eq. (45) with respect to temperature, we obtain the specific heat per mole at constant volume under CPM symmetry:

$$C_v = \frac{dU_{mol}}{dT} = \frac{d}{dT}\frac{3N_A \aleph h\nu}{e^{\frac{h\nu}{kT}} - 1} = 3\aleph N_A k \frac{\left[\frac{h\nu}{kT}\right]^2 e^{\frac{h\nu}{kT}}}{\left[e^{\frac{h\nu}{kT}} - 1\right]^2}. \tag{46}$$

The specific heat of positive or negative mass substances can be interpreted in terms of statistical mechanics. In statistical mechanics, entropy is related to the number of modes per unit volume at constant temperature. At constant volume, a body with a positive mass absorbs photons and gains entropy when in communication with a hotter environment and loses photons and entropy when in communication with a colder environment. By contrast, a body with a negative mass at constant volume loses photons and entropy when in communication with a hotter environment and gains photons and entropy when in communication with a colder environment.

In investigations on both reversible and irreversible systems composed of positive and negative mass, there is a relationship between the flow of entropy and the flow of photons. That is, the flow of entropy is coincident with the flow of photons. If the flow of photons is equivalent to the flow of entropy, then it is reasonable to conclude that the photon can be considered the basic unit of entropy. Since the number of photons per unit volume vanishes at absolute zero, the entropy differences between all states of a system must also disappear at absolute zero as Nernst stated in the third law of thermodynamics [114,115].

5. Energy of a photon

A photon can be characterized in part by its energy. However, the energy of a photon ($\aleph h\nu$) propagating in free space is undetermined since there is no substance interacting with the photon to which a value of $\aleph$ can be assigned. In principle, the energy content of a photon can be measured with a blackbody calorimetric detector made of positive mass or a blackbody calorimetric detector made of negative mass. At a given temperature, a detector made of positive mass will absorb photons from the hotter environment and become hotter, yielding a positive value for the energy of a photon $(+h\nu)$. On the other hand, a calorimetric detector made of negative mass will emit more photons to the hotter environment and become cooler, yielding a negative value for the energy of a photon $(-h\nu)$. The effective mass of the photon is given by $(m = \frac{\aleph h\nu}{c^2})$, resulting in positive energies and masses when measured with positive mass detectors, negative energies and masses when measured with negative mass detectors, and zero average energy and massless when measured equally with symmetrical detectors composed of matter or antimatter [9].

6. Entropy of a photon

In his *Treatise on Thermodynamics*, Planck [116], who was greatly influenced by Clausius [117], gave what he considered to be "the most general statement of the second law of Thermodynamics", writing, "Every physical or chemical process in nature takes place in such a way as to increase the sum of the entropies of all the bodies taking any part in the process. In the limit, i.e. for reversible processes, the sum of the entropies remains unchanged." Although Planck never accepted the description of light as a body, a full thermodynamic understanding of natural processes at the quantum level requires considering the photons that take part in natural processes as bodies that contribute to the overall change of entropy in any irreversible process.

The photon is not only a carrier of electromagnetic energy but is also a carrier of entropy. By calculating the entropy density and photon number density in blackbody radiation characterized by a given temperature, it is possible to determine that the average entropy of a single photon in a distribution of blackbody radiation is equal to $3.60k$ [118–120]. Consequently, the absorption of a photon by a system results in an increase of entropy and the emission of a photon from the system results in a decrease of entropy. While the total entropy of a system depends on the disgregation [121] or spreading [122] of the components of the system composed of positive or negative mass, irreversibility is fundamentally associated with the loss of photons from a system [60].

When a cold body of positive mass or a hot body of negative mass absorbs photons, according to the equations given above, the entropies of the bodies increase. The temperatures of the bodies increase, and/or expand or go through a phase change and become less ordered. Using historical language, the absorbed photons act as caloric. The free caloric results in a change in temperature and the bound caloric results in a change in phase. Caloric [123], which was originally described by Lavoisier [124,125] as an exquisitely elastic fluid, was reinterpreted by Clausius as entropy [126–129]. CPM symmetry suggests a photon gas is a modern interpretation of the "discredited" exquisitely elastic fluid known as caloric. It is important to know that the mechanical theory of heat, initiated by Rumford [130] and Humphry Davy [131], did not discredit the caloric theory of heat, since Rumford and Davy considered that caloric was generated by friction. This frictional heat can also be interpreted readily in terms of the electromagnetic force carrier. That is, a photon gas is produced in the frictional heating that results from motion.

The photon is characterized not only by its energy but also by its entropy [94,117]. Since the number of photons per unit volume vanishes at absolute zero, the entropy differences between all states of a system must also disappear at absolute zero consistent with the third law of thermodynamics [114,115].

7. Conclusions

There are assumed symmetries in the laws of nature [132]. The current laws of nature are based on symmetries [133] "which express the fact that the laws mentioned make no difference between left and right and that backward in time according to them is equivalent to forward in time." These symmetries are known as CPT symmetry. The arrow of time is often associated with a change of entropy. After identifying an asymmetry in the order of events in time that occurs in the reversible Carnot engine where there is no change of entropy, I explored the consequences of CPM symmetry in which mass may be positive or negative, and matter is considered to have a positive mass and antimatter is considered to have a negative mass. I found that the order of events in time that occur in a Carnot engine composed of antimatter would be opposite to those that occur in a Carnot engine composed of matter [1,9].

In this paper, CPM symmetry gives rise to versions of the Stefan–Boltzmann law, Planck's blackbody radiation law, and Einstein's equation for specific heat for positive and negative mass. In CPM symmetry, the flow of entropy is coincident with the flow of photons. If the flow of photons is equivalent to the flow of entropy, then it is reasonable to conclude that the photon may be considered to be the basic unit of entropy and at absolute zero, where the number of photons per unit volume vanishes, entropy differences must also vanish.

According to Daniel Sheehan [134], the second law of thermodynamics "has been confirmed by countless experiments and has survived scores of challenges unscathed. Arguably, it is the best tested, most central and profound physical principle crosscutting the sciences, engineering, and humanities." The second law of thermodynamics [3,9,60] in the context of CPM symmetry may help extend the profound physical principle to the understanding of antimatter at the very small and very large scales.

In 1911, at the first Solvay Congress on Physics, Marcel Brillouin [111] said, "It seems certain that from now on we will have to introduce into our physical and chemical ideas a discontinuity, something that changes in jumps, of which we had no notion at all a few years ago." Today, more than a century after the first Solvay Congress, physicists are again looking for new physics beyond the standard model (BSM). This is because the standard model of physics (SMP) is unable to explain puzzles such as the nature of dark matter, dark energy, and the origin of matter-antimatter asymmetry [135]. The new BSM physics offers supersymmetries (SUSY) as a "symmetry principle characterizing a BSM framework with an infinite number of models" to solve the puzzles [136]. As an alternative to branching out with SUSY, I have returned to the primary roots of physics to look for solutions to puzzles unanswered by the SMP. Perhaps CPM symmetry as opposed to CPT symmetry will provide a fresh way of understanding the nonluminous substances (some extremely massive stars may appear to be nonluminous as a result of an extreme gravitational redshift of the emitted radiation that is predicted independent of the theory of general relativity [137]) in the universe known as dark matter [138–141]. Indeed, the gamma ray signals considered to come from annihilating dark matter [142] and predicted by SUSY are intriguing in light of the observed annihilation of matter and antimatter consistent with CPM symmetry [3].

If CPT symmetry, based on ordinary physics and relativity and even on extended or nonrestricted special relativity to include superluminal velocities [6–8,143,144], be the true symmetry, there would be much new physics to look forward to and the analysis presented in this paper would be patently wrong. Indeed, CPT symmetry has never been shown to be violated in physics. However, if CPM be the true symmetry, there would also be new physics to come. The veracity of the each symmetry is subject to observational tests. CPM symmetry predicts that dark matter is equivalent to antimatter.

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