The Intrinsic and Contingent Properties of the Binary Photon: The Equivalence of Color, Intrinsic Wavelength, and Intrinsic Frequency in Dielectric Media

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Abstract

The wavelength of light is thought to shorten as light goes from a rarer medium (air or vacuum) to a denser medium (water or glass) and to lengthen as light goes from a denser medium to a rarer medium. Since the linear momentum of light is equal to the ratio of Planck's constant to the wavelength of light, the change of wavelength would mean that the linear momentum would increase as light goes from a rarer medium to a denser medium and decrease as light goes from a denser medium to a rarer medium. Since the light that exits a refracting medium is indistinguishable from the light that enters the refracting medium, this would be in direct conflict with the conservation of linear momentum, which otherwise is a fundamental principle of nature. Here we show, using the model of the binary photon that the intrinsic wavelength, which is equal to the circumference of the path of the semiphotons projected on the transverse plane, is invariant as it propagates through media of different refractive indices. This wavelength is related to the intrinsic and invariant energy, linear momentum, and angular momentum. The intrinsic wavelength is related to the intrinsic frequency by the dispersion relation: $\lambda\nu = c$. The binary photon is an oscillating rotor whose rotation and oscillation are invariant. The binary photon is a rotating oscillator that produces a linearly polarized electric field and a circularly polarized magnetic field that are a quarter of a wavelength out-of-phase with each other. Because of the change in velocity of the propagating invariant rotating oscillator, the electromagnetic fields contract in the direction of propagation when light propagates from a rarer to a denser medium and expand in the direction of propagation when light propagates from a denser medium to a rarer medium. The change in the wavelength in a refracting medium gives rise to the Minkowski momentum and the change in velocity in a refracting medium gives rise to the Abraham momentum. Individually the Minkowski and Abraham momenta are not conserved but the geometrical mean of these two momenta is equal to the intrinsic and conserved linear momentum. Like the Minkowski and Abraham momenta, the wavelength of the electric and magnetic fields is not an intrinsic wavelength but a contingent wavelength that depends on the refractive index of the refracting medium. The contracted and expanded fields interfere in three dimensions in the refracting medium just as they do in a vacuum. The intrinsic properties of the binary photon are sufficient to explain diffraction in a refracting medium consistent with the conservation of linear momentum. We also show that a study of diffraction in a refracting medium reveals that the binary photon has intrinsic, conserved, and invariant properties described by its intrinsic wavelength and frequency as well as reversible social properties, such as the crowding of binary photons along the axis of propagation that are described by its contingent wavelength and frequency.

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1. Introduction

Arthur Compton [1] showed that the photon can be defined in terms of its energy and linear momentum—two conserved quantities. The energy (E) of a photon is typically defined in terms of the scalar temporal quantities of frequency (ν) or angular frequency (ω):

$$E = h\nu = \hbar\omega \quad (1)$$

Since a photon that leaves a refracting medium is indistinguishable in terms of its frequency from a photon that enters the refracting medium, the idea that the frequency is invariant in a refracting medium is consistent with the conservation of energy.

The linear momentum (p) of a photon is typically defined in terms of the directional spatial quantities of wavelength (λ) or wave number (k):

$$p = \frac{h}{\lambda} = \hbar k \quad (2)$$

Since a photon that leaves a refracting medium is indistinguishable in terms of its wavelength from a photon that enters the refracting medium, the idea that the wavelength is invariant in a refracting medium is consistent with the conservation of linear momentum. However, the standard textbook treatments of refraction as well as the treatments given in more specialized texts are based solely on the assumption that energy is conserved, and they are in conflict with the conservation of linear momentum [2-4].

In *Fundamentals of Physical Optics* and *Fundamentals of Optics*, textbooks that served generations of physicists,[2] Jenkins and White [5-8] describe the change of wavelength that occurs when light enters a refracting medium: "*Passage from one medium to another causes a change in the wavelength in the same proportion as it does in the velocity, since the frequency is not altered…For since wavelengths are proportional to velocities, we have* $\frac{\lambda}{\lambda_m} = \frac{c}{v} = n$ *when the light passes from a vacuum, where it has wavelength* λ *and velocity* c*, into a medium where the corresponding quantities are* λ_m *and* v."

Elizabeth Slayter [9] and Sönke Johnsen [10], both of whom apply physics to the study of biology, also posit that the wavelength is not invariant but depends on the refractive index of the dielectric medium. In *Optical Methods in Biology*, Slayter writes, "*Frequency…is the quantity which determines the 'kind' or color of light and is thus also a quantity amenable to direct observation. Frequency is related to the wavelength* λ *by the expression* $\nu = \frac{c}{\lambda_{vac}} = \frac{c}{n_{med}\lambda_{med}}$." Likewise, in *The Optics of Light. A Biologist's Guide to Light in Nature*, Johnsen writes, *"…suppose that a beam of sunlight goes from air (n ≅ 1) into the ocean (n ≅ 1.33). The index goes up, so the phase velocity drops by a factor of 1.33. Since this is the product of the frequency and wavelength, one of the two (or both) has to also drop. It turns out that the frequency stays the same and the wavelength drops. In this case, a "green" 550 nm photon actually has a wavelength of 414 nm in the ocean. So frequency seems to be more fundamental than wavelength. Also, remember the energy of a photon is proportional to frequency, but not to wavelength. . . This is important, because in many processes, such as absorption, it is the energy of the photon that matters, not its wavelength. For example, even though the wavelength of a "green" photon inside our eye depends on whether the eye is full of water or air, our perception of it doesn't change, because absorption of light by photoreceptors depends on the energy of the photons, which is related to the unchanging frequency*."

On the other hand, in an analysis of the Snel-Descartes law for a single photon based on the conservation of both energy and linear momentum, Wayne [11] concluded that both frequency and wavelength are invariant as light propagates across an interface between two dielectrics, and consequently, energy can be equivalently defined in both the temporal and spatial domains by:

$$E = h\nu = \hbar\omega = \hbar kc = \frac{hc}{\lambda} \quad (3)$$

Likewise, linear momentum can be equivalently defined in both the spatial and temporal domains by:

$$p = \frac{h}{\lambda} = \hbar k = \frac{\hbar\omega}{c} = \frac{h\nu}{c} \quad (4)$$

Since the spatial and temporal descriptors of the photon are invariant, the frequency (ν), wavelength (λ), angular frequency (ω), and wave number (k) can be used equivalently to characterize the color of a monochromatic photon, contrary to Slayter's [9] and Johnsen's [10] conjecture that frequency is more

[2] Decades ago David Lee (https://physics.cornell.edu/david-lee) recommended this book to me.

fundamental than wavelength. Moreover, since the spatial and temporal descriptors are invariant, they can be considered to be *intrinsic* properties of a monochromatic photon. With this view, the refractive index of a dielectric slows down a photon without changing its total energy or linear momentum [11].

Treatments of the linear momentum of a photon propagating through a dielectric medium with refractive index ($n_i = \frac{c}{v_i}$) infer that the linear momentum of a photon in a dielectric either increases or decreases as it enters a dielectric [12-30]. The Abraham or kinetic momentum ($= \frac{h}{n_i \lambda}$) of a photon in a dielectric is considered to be smaller than the linear momentum of a photon in a vacuum because the velocity (v_i) of photons in a dielectric is slower than the velocity in a vacuum (c). On the other hand, the Minkowski or canonical momentum ($= n_i \frac{h}{\lambda}$) of a photon in a dielectric is considered to be greater than the linear momentum of a photon in a vacuum if the wavelength of photons in a dielectric is shorter than the wavelength in a vacuum. Attempts to clarify this unsolved controversy have resulted in "*an extensive and confusing literature*" [18, see https://en.wikipedia.org/wiki/Abraham%E2%80%93 Minkowski_controversy]. Ginzburg [31] calls the controversy one of the "*perpetual problems*" in physics. The Abraham and Minkowski momenta individually are related to aspects of the contingent and reversible properties of a photon as opposed to the intrinsic and conserved properties of a photon, while the geometrical average ($\sqrt{\frac{h}{n_i \lambda} \frac{n_i h}{\lambda}} = \frac{h}{\lambda}$) of the Abraham and Minkowski momenta is equal to the refractive-index-independent, invariant, and conserved linear momentum given above [11].

The quantum mechanical photon is typically described as being a mathematical point having four descriptive properties—frequency, wavelength, angular momentum, and velocity. The relationship states that as the photon enters a more dense material the velocity decreases so the wavelength shortens. We believe this is an incomplete description. We will show that the photon is described better as having angular momentum, velocity and two descriptive wavelengths—a traditional wavelength that we call the contingent wavelength and an additional wavelength that we call the intrinsic wavelength. There are also two frequencies—the contingent frequency and the intrinsic frequency.

These two frequencies equal each other in a vacuum. Likewise, the two wavelengths equal each other in a vacuum. The contingent wavelength shortens and the contingent frequency increases when light passes through a more dense material in the traditional sense, but the intrinsic wavelength and frequency always remain the same.

This additional intrinsic wavelength is possible if a photon is not a mathematical point, but a composite entity with extension in which two particles spiral with opposite senses. The intrinsic wavelength is defined by the circumference of the path of the two spiraling particles for one rotation projected onto the transverse plane. In a vacuum the circumference of the spiral is equal to its traditional wavelength. The radius of the projected circle is equal to the reciprocal of the wave number.

Photons in the blue light region of the spectrum have a smaller spiral than photons in the red light region of the spectrum. As each of the lights pass from one medium to the next they would slow down or speed up but each of their spiral circumferences remains the same. So the photons of blue light do not shorten to photons of ultraviolet light. The photon itself does not change; it just slows down.

By considering the *intrinsic* properties of this extended photon that we call the binary photon [32-37], it becomes natural to incorporate the conservation of linear momentum into a description of refraction [11]. By distinguishing the *intrinsic* and *invariant* properties of the binary photon from the *contingent* and *reversible* properties of the binary photon, the nature of light and its propagation through matter become more intelligible.

The binary photon is not a geometrical point-like elementary particle but a composite particle composed of conjugate particles of matter and antimatter, which Wayne [32-37] calls semiphotons. Assuming that time is unidirectional, Wayne [38-41] assumes that charge-parity-mass (CPM) symmetry is more realistic than charge-parity-time (CPT) symmetry in describing matter and antimatter. With CPM symmetry, the net charge and net mass of the binary photon in free space vanishes. As a result of the gravitational force between semiphotons of opposite mass and the Coulombic force between semiphotons with opposite charge [34], the binary photon propagates in the vacuum at a speed equal to the reciprocal of the square root of the product of the electric permittivity (ϵ_o) and the magnetic permeability (μ_o) of the vacuum (the speed of light) until it comes in contact with a dielectric, at which point the binary photon travels at a lower velocity. The velocity returns to the vacuum speed of light when the binary photon emerges from the dielectric.

As the binary photon propagates, the semiphotons rotate transversely to the axis of propagation in a manner that satisfies Sommerfeld's [42] demand that the angular momentum of the photon equals $\hbar$. The circumference of the path projected on the transverse plane is equal to the wavelength of light. As the conjugate semiphotons rotate with opposite senses around the axis of propagation, they simultaneously oscillate antisymmetrically in the axial direction in a manner that prevents them from colliding into one another [35]. The rotation and oscillation of the semiphotons result in a linearly polarized electric field and a circularly polarized magnetic field that are a quarter of a wavelength out-of-phase with each other [33,36]. The eigenvalues of the rotational and translational energy as well as the linear and angular momentum can be solved using the quantum mechanical Schrödinger equation for a boson and the classical equations of mechanics [37]. In essence, the binary photon is a quantized propagating electromagnetic rotator and oscillator with intrinsic and invariant energy, linear momentum, wavelength, and frequency. Binary photons also interfere with each other in three spatial dimensions [33]. While the intrinsic properties of the binary photon are invariant, the contingent properties vary with the refractive index of the medium.

Thomas Young [43,44], a physician, used the interference effects that accompany diffraction of light in air to determine for red, orange, yellow, green, blue, indigo, violet, and ultraviolet light, the length of an undulation in parts of an inch, the number of undulations in an inch, and the number of undulations in a second. William and Lawrence Bragg [45] also used the interference effects that accompany reflection from a crystal lattice to determine the wavelength of x-rays. Since all experiments were done in air or vacuum, where the refractive index is essentially unity, the refractive index did not appear in the equations used to relate the wavelength of light to the position of the diffracted light and the dimension of the object. For a transmission grating in a vacuum or in air, the wavelength of light can be determined with the following equation:

$$\lambda = d \sin\theta \qquad (5)$$

where θ is the angle subtended by the 0th and $\pm 1^{st}$ order diffracted light and for small angles is equal to the angular aperture [10]. d is the characteristic spatial distance responsible for diffraction as shown in Fig. 1 [46].

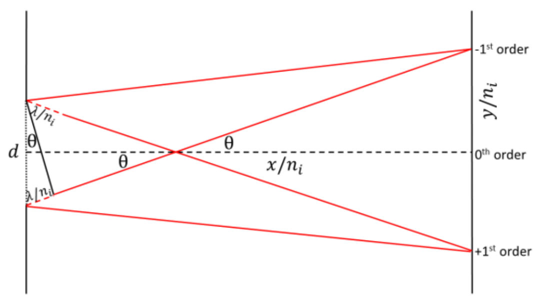


Fig. 1: The geometry of diffraction in a medium with refractive index (n_i) and intrinsic wavelength λ. Consider d to represent the width of a slit in a diffraction grating, x/n_i to represent the apparent or the refractive index-dependent distance from the diffraction grating to the observation screen, y/n_i to represent the apparent or refractive index-dependent distance between the 0th order diffraction spot and the $\pm 1^{st}$ order diffraction spot, λ/n_i to represent the apparent or refractive index-dependent difference in the distance between light diffracted from the two sides of a slit in the diffraction grating, and θ to be the angle defined by arcsin$\frac{\lambda/n_i}{d}$. The three θ's shown are equal by similar triangles and vertical angles. As long as x/n_i >> d and θ is small, $\sin\theta \cong \tan\theta = \frac{y}{x}$. Assume that the distance between the diffraction grating and the screen is so large, that $^{x}/_{n_i}$ is a good approximation of that distance. The greater the refractive index, the smaller are $\frac{x}{n_i}$ and $\frac{y}{n_i}$ and the closer to the Parafilm screen the diffraction grating appears.

Ernst Abbe, a physicist and social philosopher who was interested in designing microscope objectives that would capture the light diffracted by an object, discovered the necessity of including the refractive index in the diffraction equation [46,47]. Abbe realized that to describe image formation in a microscope, the refractive index (n) must no longer be an outsider in the diffraction equation. Abbe's diffraction equation is:

$$\lambda = n\, d \sin\theta \qquad (6)$$

In fact, Abbe defined the numerical aperture (NA) of an objective lens, the most important characteristic of an objective lens, to be equal to $n \sin\theta$, which is the product of the refractive index and the angular aperture [9].

By combining Abbe's law with the textbook equation [2-8] that gives the relationship between wavelength and refractive index without taking conservation of linear momentum into consideration, we get the following equation for the wavelength of diffracted light in a refracting medium (λ_{med}):

$$\lambda_{med} = \frac{\lambda}{n} = d \sin\theta \qquad (7)$$

On the other hand, if we combine Abbe's law with the equation given by Wayne [11] that takes the conservation of linear momentum into consideration, we get the following equation for the wavelength of light in a refracting medium [48,49]:

$$\lambda_{med} = \lambda = nd \sin\theta \qquad (8)$$

We performed the experiments described in this paper to determine how the refractive index of the medium affects the properties of diffracted light in terms of the wavelength of light. The experiments led us to distinguish between the intrinsic wavelength and the contingent wavelength of the binary photon.

2. Materials and Methods

The diffraction chamber was made from the upper portion of a 9.3 cm × 9.3 cm × 8.89 cm baseball display case (model 97411; Darice, Inc., Strongsville, OH, USA) with an inside length of 7.6 cm. The laser light was produced by a 632.8 nm neon-helium laser (model ML820; Metrologic Instruments Inc., Bellmawr, NJ, USA). The laser light was diffracted by a 500,000 lines per meter "teaching" diffraction grating that had been removed from a 2" × 2" cardboard slide holder and fastened to the inside surface of the front wall of the diffraction chamber. The diffraction pattern was observed and measured with a compass on the inside surface of a Parafilm (American National Can, Neenah, WI, USA) screen placed against the inside surface of the back wall of the diffraction chamber. The distance between the 1st order diffraction spots was measured with a compass.

The filters were placed parallel and adjacent to the diffraction grating. The Kodak Wratten filters, originally used to observe and photograph chromosomes, were inherited from Lester W. Sharp (Cornell University) and the model 10LF10-633-B 632.8 $\pm$ 2 nm, 10 $\pm$ 2 nm FWHM interference filter was purchased from Newport Corporation (Irvine, CA, USA).

Photographs of the diffraction patterns were taken using a Nikon D750 digital single lens reflex (DSLR) camera body. The Nikon D750 DSLR camera body utilizes a 24.3 megapixel, 36mm x 24mm full frame CMOS sensor and allows for manual toggling of autofocus capabilities – essential for evaluation of virtual image distances during comparative analysis of photographs. The camera body was affixed with a Nikon AF FX NIKKOR 50mm f/1.8 lens with autofocus disabled, and all photographs used in data analysis were taken at f/1.8. The affixed lens was not equipped with a lens filter, protective or otherwise, and hands-free shutter actuation was utilized at all times in order to preserve image uniformity and ease of image processing. The camera body was tripod-mounted throughout the duration of the experiment, and any movement or play between connections was minimized before photographing began. We captured the first image in each experiment using the autofocus sensor module with 51 points of detection, with through-the-lens (TTL) phase detection and fine-tuning of image sharpness. Once focused using onboard autofocus technology the camera body's autofocus motor was manually disabled by moving the autofocus switch near the camera's bayonet mount to the "M" (for "manual") position, ensuring further changes in focal length could only be attained through rotation of the focus ring on the affixed 50mm lens, which remained stationary throughout the data collection process. This process of focusing the image was repeated at the onset of every trial, and inadvertent changes in focal length through rotation of the focus ring would result in the immediate termination of data collection for that trial, requiring the round to be repeated.

Photographs of the gnome were taken with an XtremePro Sports CAM 1080P HD edition underwater camera (https://images-na.ssl-images-amazon.com/images/I/A15BNDAtCfL.pdf).

Experiments were performed at room temperature (23-26 C).

3. Results and Discussion

When light is diffracted in air by a grating composed of 2 μm slits a diffraction pattern is observed (Fig. 2a). When the air is replaced by water, the distance between the two 1st order fringes contracts in a way that is consistent with Abbe's law (Fig 2b). The color of the spots was the same whether observed in air or in water.

To determine $\sin\theta$, we used a compass to measure the distance between the two 1st order spots on the Parafilm screen mounted on the inside surface of the diffraction chamber and divided by two to obtain the length of the side opposite the angle. The side adjacent to the angle was 7.6 cm. We used the Pythagorean theorem to calculate the hypotenuse. The sine of the diffraction angle is equal to the ratio of the length of the opposite side to the length of the hypotenuse.

The validity of Eqns. (7) and (8) was tested by inserting a given filter in the air and water between the diffraction grating and the Parafilm screen. If the wavelength in water (λ_{med}) was reduced by the refractive index of water ($n = 1.333$) from the wavelength in air (λ), then the wavelength determined by diffraction would be given by:

$$\lambda_{med} = \frac{\lambda}{n} = 475 \text{ nm} \tag{9}$$

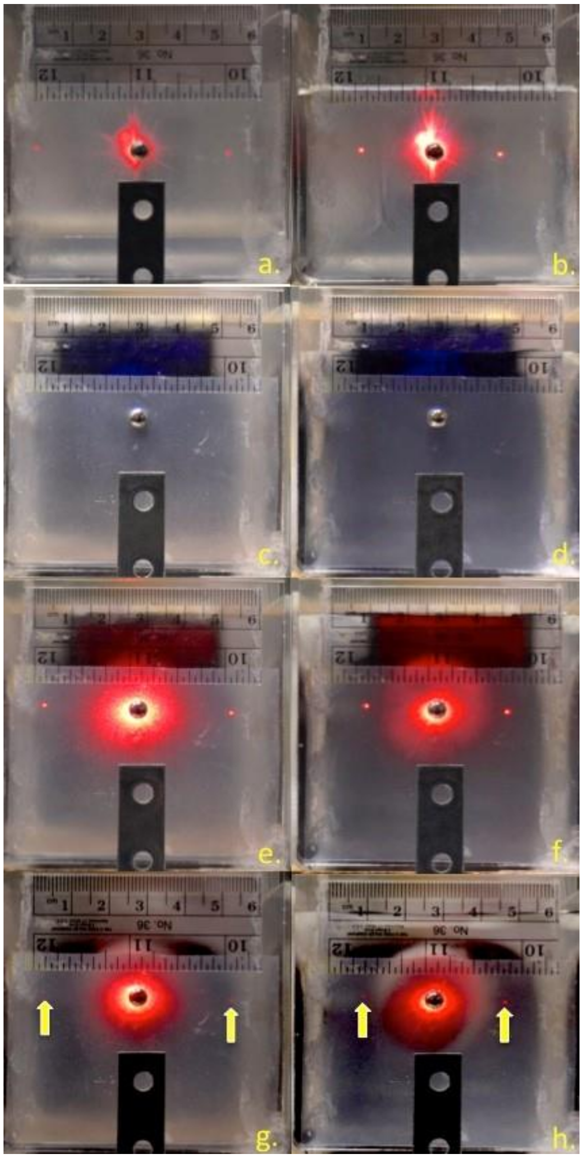


Fig. 2. Photographs of the diffraction pattern produced in air and water without a filter (a and b, respectively), with a Kodak Wratten # 47 blue filter (c and d, respectively), with a Kodak Wratten # 29 red filter (e and f, respectively), and with a 632.8 nm laser line interference filter (g and h, respectively). Arrows indicate the positions of the faint spots observed with the interference filter. The photographs, which illustrate the essence of the observed effect, include an additional refraction due to light propagating through the translucent back wall of the chamber. Measurements were made on the inside wall of the chamber.

This wavelength is in the blue region of the spectrum and if present should be transmitted by a blue Kodak Wratten # 47 filter (Fig. 3). However no laser light was transmitted through the blue Kodak Wratten # 47 filter in either air or water (Figs. 2c and 2d). By contrast, the laser light was transmitted through a red Kodak Wratten #29 filter in both air and water (Figs. 2e and 2f), indicating that the wavelength and color did not change. This confirms John Tyndall's [50] statement that "*the Color of Light is determined solely by its Wave-length* [and not by refraction]."

	No Filter	Red Kodak Wratten filter	Red Interference Filter	Blue Kodak Wratten Filter
n	1.3381 ±0.0000	1.3212 ±0.0000	1.3239 ±0.0008	–

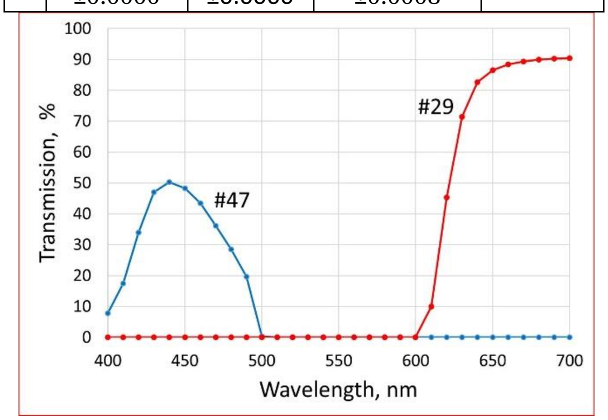


Fig. 3: Transmission spectra of the Kodak Wratten filters used in these experiments [51].

Since Kodak Wratten filters are typically considered to be absorption filters that may transmit light based on its frequency, we also tested a 632.8 nm laser line interference filter. Interference filters are typically considered to reflect or transmit light based on its wavelength. The laser light was transmitted through this filter in both air and water (Figs. 2g and 2h), indicating that there was no significant change in wavelength when photons propagate through media with different refractive indices.

Since we found that the color and wavelength did not depend upon whether light propagated through air or water, we could use Eqn. (8) to measure the refractive index of water. When Eqn. (8) is written explicitly for air and water, we get:

$$\lambda = n_{air}\, d\, \sin\theta_{air} \tag{10}$$

$$\lambda = n_{water}\, d\, \sin\theta_{water} \tag{11}$$

By combining Eqns. (10) and (11), and assuming that the wavelength and slit width are invariant, we get the Snel-Descartes law:

$$n_{air}\ \sin\theta_{air} = n_{water}\ \sin\theta_{water} \tag{12}$$

Since the refractive index of air is unity, the refractive index of water can be determined with the following equation:

$$n_{water} = \frac{\sin\theta_{air}}{\sin\theta_{water}} \tag{13}$$

We determined the refractive index of water to be 1.3381 (Table 1), consistent with the value measured with an Abbe refractometer (1.3321). With the red absorption and interference filters, the apparent refractive index of water was measured to be slightly lower than that measured without a filter. We were unable to measure the refractive index of water using the blue Kodak Wratten filter since no light passed through it.

Table 1. The refractive index (*n*) of water measured by diffraction. No diffraction pattern was observed when the blue Kodak Wratten filter was inserted between the diffraction grating and the Parafilm screen in either air or water. Data are present as $\bar{x} \pm$S. D. (5 replicates).

In order to further investigate the effect of refractive index on the diffraction of light, we tested the effect of the three filters on the diffraction of light in various (0%, 10% (w/v), 20% (w/v), and 30% (w/v)) solutions of sucrose. Again we found that without a filter, the refractive index measured by diffraction was similar to the refractive index measured with the Abbe refractometer (Fig. 4) and with published results [52]. We also confirmed that with the red absorption and interference filters, the apparent refractive index of water was measured to be slightly lower than that measured without a filter. The fact the filters pass light with the incident wavelength and not light with the incident wavelength divided by the refractive index means that we should search for an explanation of the difference in the measured refractive index when using the red filters.

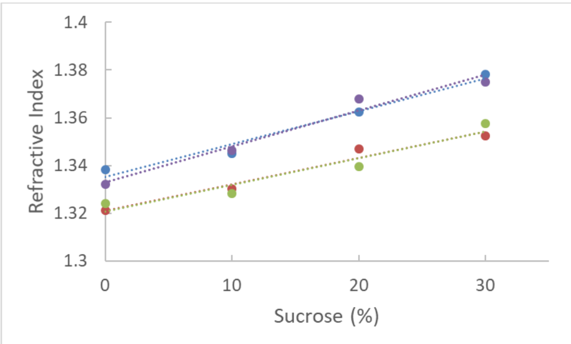


Fig. 4: The effect of sucrose concentration (% w/v) on the refractive index (y). The regression equations are a) $y = 0.0015x + 1.3328$ (r^2=0.9680) for the Abbe refractometer, ●; b) $y = 0.0014x + 1.3353$ (r^2=0.9718) without a filter, ●; c) $y = 0.0011x + 1.3211$ (r^2=0.96408) with the Wratten #29 filter, ●; and d) $y = 0.0011x + 1.3205$ (r^2=0.9324) with the Newport interference filter, ●. Averages of five replicates.

Both the red absorption and red interference filters are constructed using refractive layers. Consequently, the light diffracted by the grating is not only refracted by the air and water, but also it is differentially refracted by the air-filter interfaces and the water-filter interfaces. The effect of a filter placed in either air and water results in an image of the slit that appears closer (Fig. 5). Of importance here is that the image appears to be closer when there is an air-filter interface than when there is a water-filter interface. This is because the refractive power of the filter is greater for non axial light at an air-filter interface than at a water-filter interface. The effect of filters on shifting the focus is well known to astronomers [53] working with the Hubble telescope.

The paraxial focal shift (fs) is given by:

$$fs = t\frac{(n_{filter} - n_{medium})}{n_{filter}} \quad (14)$$

where t is the thickness of the filter with refractive index n_{filter}.

Since there is a greater focal shift when the filters are used in air than when the filters are used in water, the apparent refractive index of water measured with the filters is less than the refractive index of water measured without a filter. This is consistent with Eqn. (13). The refractive index of water for a given wavelength of light is not a constant but a coefficient that depends on the measuring conditions, including temperature and pressure [54-56].

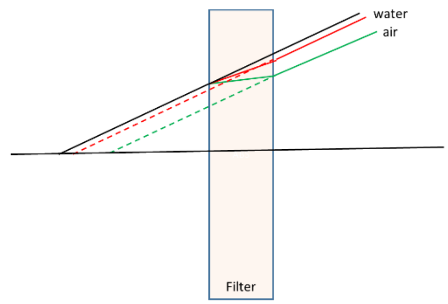
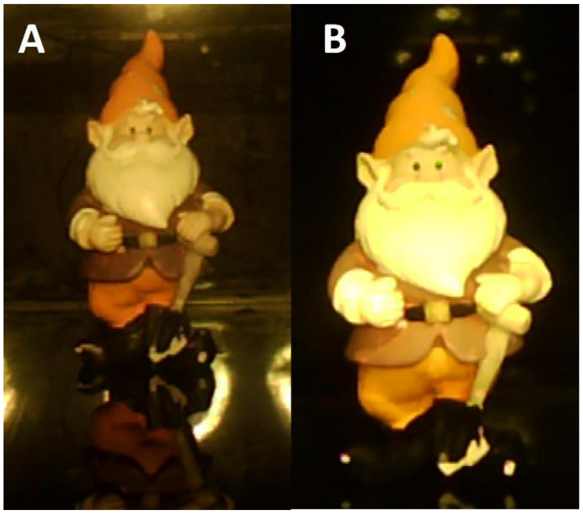


Fig. 5: The effect of a filter on the refraction of light. In the presence of a filter, the parallel sides result in a displacement of the incident light. The displacement is greater at the air-filter interface (green) than at the water-filter interface (red). The black line represents the path of light in the absence of a filter.

We used the filters to show that the wavelength of light is invariant whether it is propagating in air or water. If the wavelength does not decrease in water, why does the distance between the 1st order spots decrease? The distance decreases because the water bends or refracts the light exiting the diffraction grating in a refractive index-dependent manner towards the 0th order ray that is perpendicular or normal to the slit. The distance (y) between the 1st order spot and the normal to the slit is inversely



proportional to the refractive index (n_i). Thus when the wavelength is invariant and linear momentum is conserved, the correct trigonometric presentation of the relationship between the distance of the 1st order spot and the normal must include the refractive index as shown in Fig. 1.

When the diffraction grating is viewed and measured in water, it seems as if the light from the slit diverges from a point (W) that is closer to the Parafilm screen than when the grating is viewed and measured in air, where it seems as if the light from the slit diverges from a point (A). This interpretation is illustrated in Fig. 6.

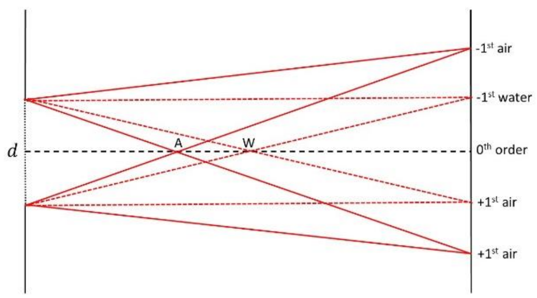


Fig. 6: When a diffraction grating is viewed in air (solid lines) the light seems to diverge from a slit at position A. When a diffraction grating is viewed in water, the light is bent toward the normal and the light seems to diverge from a slit at position W. That is, the angle that the light enters the eye or camera is greater in water than in air. As a result, the slit seems to be closer in water than in air. *d* is the width of the slit.

To demonstrate conclusively that the refracting medium functions to bend the light rather than to change the wavelength of light, we photographed an object using an XtremePro underwater camera and used ImageJ to measure the relative distance between two predetermined points. The relative distance between the points was 1.333 ± 0.016 ($\bar{x} \pm$ S. D) times greater when the object was photographed in water compared to when it was photographed in air (Fig. 7).

Fig. 7: (A) The photograph of the gnome was taken in air; (B) The photograph of the gnome was taken under the same conditions in water. The image of the gnome in water *appears* $1.331 \pm 0.013(7)$ times larger or 1.331 times closer to the camera.

The refractive index of a medium can be defined as the ratio of the actual distance to the apparent distance in the medium:

$$n = \frac{\text{actual distance}}{\text{apparent distance}} \tag{15a}$$

or

$$\text{apparent distance} = \frac{\text{actual distance}}{n} \tag{15b}$$

This relationship is well known to undersea divers [57].

When the conservation of linear momentum is taken into consideration when studying diffraction in a refracting medium, we see that the index of refraction plays a role in bending the paths of the photons rather than changing their wavelength. This is consistent with Newton's conclusions. Newton [58,59] showed that "*when the Rays which differ in Refrangibility are separated from one another, and any one Sort of them is considered apart, the Colour of the Light which they compose cannot be changed by any Refraction or Reflexion whatever, as it ought to be were Colours nothing else than Modifications of light caused by Refractions, and Reflexions, and Shadows*." This result was the foundation of Newton's Prop. II Theor. II, which states: "*All homogeneal Light has its proper Colour answering to its Degree of Refrangibility, and that Colour cannot be changed by Reflexions and Refractions*."

John Herschel [60] concurred "*that between these two qualities—refrangibility and colour—an absolute and invariable connexion exists*." After explaining some apparent discrepancies, Herschel wrote, "*And hence we conclude that colour is not a superinduced but an inherent quality of the luminous rays*."

With Newton and Herschel, we will define the color of monochromatic light by its refrangibility with or without diffraction. In the absence of optical illusions [61,62] or colorblindness [63], the identity and naming of this color will be agreed upon by most trichromatic observers.

The experiments presented in this paper give us reason to consider that the color as well as the wavelength and linear momentum and frequency and energy are intrinsic properties of the photon that are invariant and conserved as a photon propagates across an interface. This is intelligible in terms of the binary photon where the circumference is equal to the wavelength of light and directly related to the energy, linear momentum, and angular momentum [23,25].

Light can be considered to be a mechanical system [1] where a mechanical system is defined by its energy and momentum. The wave functions that describe the rotatory and oscillatory movements of the semiphotons that make up the binary photon have been constrained by the energy and the momentum of the binary photon they describe [32]. The wave functions that describe the three-dimensional paths around the center of gravity along which the leading and following semiphotons move are given in Eqns. (16a) and (16b). Eqn. (16a) describes the path of the leading semiphoton in a Cartesian coordinate system:

$$\Psi^{leading-position} = \begin{bmatrix} \frac{\lambda}{2\pi}\cos[\theta] \\ P\frac{\lambda}{2\pi}\sin[\theta] \\ \frac{c}{n_i}t + \frac{2\lambda}{(2\pi)^2}\cos^2[\theta] \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \quad (16a)$$

where $\theta = 2\pi w$ is the refractive index-independent argument, P represents the parity of the semiphoton and is +1 for an anticlockwise rotation when looking at the source and -1 for a clockwise rotation, and w represents the phase of the binary photon, which varies between 0 and 1. The terms with a cosine or sine represent the rotational properties of the binary photon, and the term that contain a cosine squared represents the oscillatory properties of the binary photon. The terms with λ represent the *intrinsic* properties of the binary photon, and $\frac{c}{n_i}t$ term, which does not include λ, is the only term that is influenced by the refractive index (n_i). This term reduces the wavelength and increases the frequency of the electric and magnetic fields [33] in a reversible manner *contingent* upon the refractive index with respect to a given wavelength. Eqn. (16b) describes the path of the following semiphoton:

$$\Psi^{following-position} = \begin{bmatrix} \frac{\lambda}{2\pi}\cos[\theta] \\ -P\frac{\lambda}{2\pi}\sin[\theta] \\ \frac{c}{n_i}t - \frac{2\lambda}{(2\pi)^2}\cos^2[\theta] \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \quad (16b)$$

and the terms describe the same properties as they do in Eqn. (16a).

The intrinsic wavelength of the binary photon, which is inversely proportional to the energy of the binary photon, is equal to the length of the paths of each semiphoton projected on the plane perpendicular to the axis of propagation [35]. These paths are equivalent to the circumference of the binary photon. The wave functions describe paths in Euclidean space and Newtonian time along which the movement of the semiphotons can be visualized [35]. These wave functions give energy and momentum eigenvalues when operated upon by the Schrödinger equation for a boson and the classical equations of physics [37]. The binary photon propagates at velocity $\frac{c}{n_i}$ over a distance $\frac{c}{n_i}t$ as a result of the gravitational force and the electromagnetic force [34].

Here we see that the model of the binary photon, which is composed of two semiphotons that move in a three-dimensional wave-like motion helps to understand how light can be viewed as a duality—real photons that are both corpuscular and wave-like as opposed to a complementarity—virtual photons that only become existent and real when they are measured by a device that measures either their corpuscular or wave-like properties [64-74].

The binary photon thus has characteristics of a three-dimensional wave with intrinsic wavelength and frequency and a corpuscle with extension within which these waves exist. The binary photon is a rotating oscillator with constant energy, linear momentum, and angular momentum that produces a linear oscillating transverse electric field and a circular oscillating magnetic field that are a quarter of a wavelength out-of-phase with each other as the binary photon propagates. Because the intrinsic rotary oscillator propagates at a velocity that is inversely proportional to the refractive index, the wavelength of the electric and magnetic fields decreases and the frequency of these fields increases. The wavelength of the electromagnetic fields is the contingent and reversible wavelength that is dependent on the refractive index. The decreased velocity gives rise to the Abraham momentum and the decreased wavelength gives rise to the Minkowski momentum. Assuming that both effects occur, the geometric mean of the two momenta is equal to the intrinsic momentum.

The intrinsic energy and linear momentum (or the geometric mean of the Abraham and Minkowski momenta) of the binary photon are given in Eqns. (3) and (4). The contingent wavelength ($\lambda_{contingent}$) can be defined in terms of the intrinsic energy and intrinsic linear momentum of a binary photon by:

$$E =$$

$$\frac{hc}{\lambda_{intrinsic}} = \frac{hn_{med}v_{med}}{\lambda_{intrinsic}} = \frac{hv_{med}}{\lambda_{intrinsic}/n_{med}} = \frac{hv_{med}}{\lambda_{contingent}} \tag{17a}$$

$$p = \frac{E}{c} = \frac{h}{\lambda_{intrinsic}} = \frac{h}{n_{med}\lambda_{contingent}} \tag{17b}$$

The contingent wavelength does not represent the intrinsic properties of light but a relative and reversible social property that describes in addition to the wavelength of the electromagnetic fields, the linear photon density or the degree of crowding of the binary photons along the axis of propagation. The change in the velocity results in a crowding or increased linear population density of the binary photons with an invariant intrinsic wavelength along the axis of propagation as they enter a refracting medium and a decongestion or decreased linear population density as they exit a refracting medium (Fig. 8). In two different refracting media, there will be an invariant number of binary photons in a beam if the optical path lengths ($\cancel{O}PL$), which is given by the product of the refractive index and the geometrical distance, are the same. In an analysis of refraction of a single photon, Wayne [11] concluded that the optical path length is more fundamental than the geometrical length is understanding the properties of light in different refracting media. Here we see that the number of binary photons, and thus the intrinsic energy and linear momentum, in a beam propagating through equal optical path lengths are equal. The actual number of binary photons, the total intrinsic energy, and the intrinsic linear momentum will be inversely proportional to the intrinsic wavelength of the binary photons.

In the case of diffraction in a refracting medium such as water, there is a greater number of binary photons in a given geometrical distance compared with air. The apparent position of the grating that gives rise to the interference depends on the refractive index. However, the refractive index is a ratio of velocities and the square of the refractive index can be represented by the inverse of the relativistic symbol β^2 that equals $\frac{v_i^2}{c^2}$. Consequently, the apparent distance, as opposed to the geometrical distance of the grating to the screen is given by:

$$x/n_i = \frac{OPL_i}{n_i^2} = OPL_i\frac{v_i^2}{c^2} = OPL_i\beta^2 \tag{18}$$

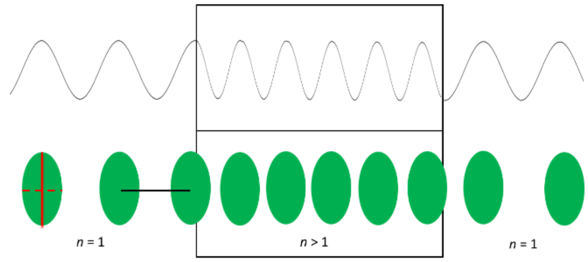


Fig. 8: A corpuscular representation of the binary photon shows that as the binary photons enter and leave a refractive medium (n > 1), the intrinsic wavelength (λ) that is represented by the major circumference of the oblate binary photons (—) is invariant. The maximal length ($\frac{\lambda}{\pi^2}$) along the axis of propagation (---) is also intrinsic and invariant. The center-center distance between binary photons along the axis of propagation ( ) represents the contingent wavelength that is refractive index dependent. The contingent wavelength is equal to the intrinsic wavelength when n = 1. The refractive index-dependent change in the contingent wavelength is shown in a wave representation of the binary photon at the top of the figure. The corpuscle represents the three-dimensional space in which the two semiphotons rotate and oscillate as the binary photon propagates.

The number (N) of binary photons in a vacuum, where $n = 1$, along the axis of propagation per given geometrical length (ℓ) can be determined from the maximal length ($\frac{4\lambda}{(2\pi)^2}$) of a binary photon parallel to the axis of propagation [33,35]:

$$\frac{N}{\ell} = \frac{\pi^2}{\lambda} \tag{19}$$

The number of binary photons in a medium of refractive index (n_i) per optical path length (OPL_i) in a line is given by:

$$\frac{N}{OPL_i} = \frac{\pi^2}{\lambda} \tag{20}$$

The ratio $\frac{N}{OPL_i}$is a constant that is equal to only the intrinsic properties of the binary photon. That is, the number of binary photons per unit optical path length is a constant that depends only of the intrinsic and invariant wavelength of the binary photon. This means that the energy and linear momentum of a given beam of light will be equal in segments of equal optical path length.

In general, in equal segments of optical path, for beams with equal photon flux density, binary photons in the blue range, will be smaller and closer together than binary photons in the green range which will smaller and closer together than binary photons in the red range. Since $OPL_i = n_i\ell$, we get:

$$\frac{N}{\ell} = n_i \frac{\pi^2}{\lambda} \tag{21}$$

The ratio $\frac{N}{\ell}$ is contingent and relative and depends on the refractive index of the medium as well as the invariant wavelength of the binary photon. Thus, the number of binary photons that make up a line of binary photons in a ray from the slit to the screen is a contingent property that depends on the refractive index of the medium.

Since the electromagnetic fields that result from the intrinsic rotary-oscillators interfere in three dimensions in a refracting medium the same way they do in a vacuum or in air [33], the binary photons that are diffracted by a grating in a refracting medium will interfere in the same way they do in a vacuum or in air, but at the new positions that result from the bending of the paths of the binary photons in a refracting medium. Moreover, all this is done in a manner where the color and intrinsic wavelength is invariant and linear momentum is conserved.

The steady state analysis of the number of binary photons with a *contingent* wavelength and frequency propagating through static elements in a given unit of space at a given time can be extended to consider the number of binary photons with an *intrinsic* wavelength and frequency present in the space sampled by a moving body over a duration of time. This extension is useful in analyzing the relativistic Doppler effect. According to Wayne [75], the relativistic Doppler effect results in radiation friction that causes a counterforce that prevents particles with a charge and/or a magnetic moment from going faster than the speed of light. In his analysis, Wayne considered the temperature-dependent linear photon density to be invariant in the front and back of a moving body at a given time but the linear momentum of the binary photons to be greater in the front than the back of a moving body due to the wavelength and frequency shifts that are described by the relativistic Doppler effect. The differential between the linear momentum of the binary photons that strike the front of a moving body and the binary photons that strike the back of a moving body provides a velocity-dependent counterforce that prevents the body from moving faster than the speed of light. This explanation utilized what we have defined here as the contingent wavelength and frequency.

We can also explain the cause of the radiation friction that results in a counterforce in a given duration of time in the space sampled by a moving body using the intrinsic wavelength and frequency presented here. With this view, the counterforce is produced over the space sampled by the moving body in a given duration of time because the linear photon density relative to a moving body appears greater in the front compared with the back of a moving body. This results in a greater number of collisions that transfer an invariant linear momentum from binary photons that strike the front of a moving body and push it backwards compared the number of collisions that transfer an invariant linear momentum from binary photons that strike the back of a moving body and push it forwards. The concept that light itself prevents a moving body from exceeding the speed of light is intelligible from the perspective of both the intrinsic and contingent wavelengths.

Table 2: Summary of the Wavelength- and Frequency-dependent Properties of the Binary Photon

Property	Intrinsic Wavelength (λ) and Frequency (ν)	Contingent Wavelength and Frequency
Energy	✓	
Linear Momentum	✓	
Angular Momentum	✓	
Color	✓	
Electromagnetic Fields		✓
Linear Photon Density (Crowding)		✓

The intrinsic wavelength is related to the intrinsic frequency by the dispersion relation: $\lambda\nu = c = v_i n_i$.

4. Conclusion

The binary photon has been useful in providing a mechanical basis for understanding special and general relativistic phenomena, including why charged particles cannot exceed the speed of light [75], the equivalence of mass and energy [76], the acceleration of the universe [77], the precession of the perihelion of Mercury [78], and the deflection of starlight [79]. It was said of George Stokes [80], "*that if you gave Stokes the Sun there was no experiment he could not do for two-pence*." Here we have demonstrated, using simple equipment [81], that diffraction in a refracting medium reveals that the binary photon has properties that are correlated with the intrinsic color of monochromatic light, including the intrinsic energy, the intrinsic linear momentum, the intrinsic angular momentum, the intrinsic wavelength, and the intrinsic frequency. These properties describe the invariant and conserved nature of light. Diffraction in a refracting medium also reveals that the binary photons also have social properties such as the wavelength and frequency of the electromagnetic fields, the linear photon density, and the degree of crowding that describe the contingent and reversible properties of light (Table 2).

Distinguishing the intrinsic and contingent properties allows one to look at the Minkowski and Abraham momenta as two contingent and reversible aspects of light whose geometrical mean is equal to the intrinsic linear momentum. In doing so, the Abraham-Minkowski controversy, one of the perpetual problems in physics [31], becomes intelligible.

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