The Kirchhoff Diffraction Equation based on the Electromagnetic Properties of the Binary Photon

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Kirchhoff's diffraction equation, which has been used for over a century to design optical instruments, exactly describes observed optical phenomena. This is a curious fact given that the derivation of Kirchhoff's equation from the Helmholtz equation requires that the Dirichlet and Neumann boundary conditions are satisfied for an arbitrary surface simultaneously in ordinary space. According to Sommerfeld and Poincaré, this should be impossible if light is an electromagnetic wave as described by Maxwell where the magnetic and electrical fields are in phase. Given Maxwell's theory of light as an electromagnetic wave, the Dirichlet and Neumann boundary conditions could be satisfied only if light vanished identically in the image space, which is clearly contrary to experience. By contrast, the Dirichlet and Neumann boundary conditions are satisfied simultaneously by the binary photon, which is composed of electrical and magnetic fields that are out-of-phase by a quarter of a wavelength. Consequently, one field satisfies the Dirichlet boundary condition while the other field simultaneously and in ordinary space satisfies the Neumann boundary condition. A complex plane wave where the magnetic and electrical fields are a quadrature out-of-phase is also a solution to the standard and a relativistic form of Maxwell's wave equation.

To derive the scalar Kirchhoff diffraction integral from the binary photon, I have developed two functions U and G that are based on the magnetic and electrical properties of light, respectively. The two functions are twice differentiable. I have obtained their normal derivatives $\frac{\partial U}{\partial n}$ and $\frac{\partial G}{\partial n}$ with the aid of Faraday's law and the Ampere-Maxwell law, respectively. In this way, $U\frac{\partial G}{\partial n}$ describes the magnetic component of the binary photon as it propagates from the source to the diffraction plane and $G\frac{\partial U}{\partial n}$ describes the electrical component. Together $U\frac{\partial G}{\partial n}$ and $G\frac{\partial U}{\partial n}$, which are a quadrature out-of-phase with each other, simultaneously satisfy the Dirichlet and Neumann boundary conditions for an arbitrary surface in ordinary space, without light vanishing identically in the image plane. $U\frac{\partial G}{\partial n}$ and $G\frac{\partial U}{\partial n}$ are both continuous across the opening of an aperture and both vanish at an opaque boundary. As a result of the properties of the binary photon, the boundary conditions that form the basis of the Kirchhoff diffraction equation and the approximations that describe Fresnel and Fraunhofer diffraction do not lead to complete darkness at the image plane but to the observed diffraction patterns in ordinary space. The binary photon is also useful in understanding three-dimensional diffraction patterns and the observed difference between lateral and axial resolution.

1. Introduction

George Green [1] developed a theorem that related the electrical potential on an arbitrary surface to the charge density within the surface. The theorem required a function (U), with physical properties that represented the sum of all of the electric charges acting on a given point divided by the distances (r) between the charges and the arbitrary point. The theorem also required a second function ($G = \frac{1}{r}$; where r is the radial distance), known as Green's function, that was exclusively geometrical. Green's theorem (Eqn. 1), which relates a surface (S) to a volume (V), requires that the two functions (U and G) and their normal derivatives ($\frac{\partial U}{\partial n}$ and $\frac{\partial G}{\partial n}$) are differentiable.

$$\iiint (U\nabla^2 G - G\nabla^2 U)\, dV = \iint \left(U\frac{\partial G}{\partial n} - G\frac{\partial U}{\partial n}\right) dS \qquad (1)$$

Green's theorem is a powerful mathematical method that relates the properties of point sources to the properties of a point on an arbitrary but convenient surrounding surface. Because Green's theorem was not published in the elite scientific journals, its power remained hidden during Green's lifetime.

Green's work was discovered by William Thomson, who subsequently shared Green's work with George Gabriel Stokes [2] and James Clerk Maxwell [3]. Thomson also republished Green's work in Crelle's Journal in 1850, 1852 and 1854 - years after Green's death in 1841 [4-7]. Kelvin also shared Green's work with Hermann Helmholtz [8], who used Green's theorem to model the tones emitted by organ pipes. Because Helmholtz was modeling waves rather than potentials, his second function ($G = \frac{\cos \boldsymbol{k}\cdot\boldsymbol{r}}{r} - \frac{1}{r}$; where $\boldsymbol{k}$ is the wave vector and its magnitude k represents the wave number) was periodic and vanished at $r = 0$. Gustav Kirchhoff [9, 10] applied Helmholtz's work to the phenomenon of optical diffraction in order to find a diffraction integral that represented the exact sum of all point sources of light on a given point in a diffraction pattern on the other side of the aperture, given the distances between each point source and the point in the diffraction pattern.

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Kirchhoff's diffraction integral, which describes the relation between the object and the image, is the standard equation used for instrument design and image analysis [11-22].

The use of Green's theorem demands that the two functions are continuous across an arbitrary surface. Poincaré [23] and Sommerfeld [24, 25] realized that Kirchhoff's diffraction integral although exact, lacked rigor in that, if light were composed of electromagnetic waves, where the electric and magnetic fields were in-phase, as postulated by Maxwell [26], then light would vanish identically on the other side of an arbitrary surface in which its value (Dirichlet boundary condition) and its normal derivative (Neumann boundary condition) simultaneously vanished [27]. Since Maxwell's treatment of electromagnetic waves was considered infallible, this presented a conundrum given that all light does not vanish at an aperture. Sommerfeld [24, 25] and his student Carslaw [28] used a mathematical trick to overcome the problem of the contradictory boundary conditions for a sinusoidal electromagnetic wave, and made the solution mathematically rigorous using the method of images and a Riemann double-space where they postulated that every point source in physical space was accompanied by a mirror image of the point source in imaginary space. While the sleight-of-hand treatment is mathematically rigorous, and the Dirichlet and Neumann boundary conditions are met simultaneously by the waves from the real and imaginary point sources, the physical assumptions of imaginary space are fanciful. Hans Bethe [29], another student of Sommerfeld's, satisfied the boundary conditions by postulating the existence of a magnetic monopole that circulates around the aperture as if it caused a magnetic dipole. Again, while this solution is mathematically rigorous, it has no physical meaning unless magnetic monopoles exist at the aperture. More recently, Miller [30] solved the problem by treating a spherical point sources as a dipolar source. All these solutions have one thing in common—they bring *twoness* to the solution.

The binary photon, which has been valuable in describing why charged particles cannot exceed the speed of light and in describing the deflection of starlight in terms of Euclidean space and Newtonian time [31-33] provides a natural solution to the problem of twoness necessary to unify electromagnetic theory with diffraction theory in a mathematically rigorous and physically meaningful manner. In the binary photon, the amplitudes of the electric and magnetic fields are orthogonal and a quadrature out-of-phase [33]. Here I propose that $U\frac{\partial G}{\partial \boldsymbol{n}}$ represents the magnetic property of light given by a cosine function and $G\frac{\partial U}{\partial \boldsymbol{n}}$ represents the electric property of light given by a sine function so that $U\frac{\partial G}{\partial \boldsymbol{n}}$ and $G\frac{\partial U}{\partial \boldsymbol{n}}$ satisfy the Dirichlet and Neumann boundary conditions at any arbitrary surface simultaneously. Thus by assuming that the binary photon is composed of equal and opposite charges, and consequently, the total energy is composed of magnetic and electrical components that are out-of-phase by a quadrature, the real observed phenomena can be described exactly and explained physically while the Dirichlet and Neumann boundary conditions are rigorously met.

Here I will present a) the derivation of the Kirchhoff diffraction integral based on the binary photon; b) the Fresnel and Fraunhofer approximations for a two-dimensional diffraction pattern; c) the three-dimensional diffraction pattern based on the binary photon; and d) the complex plane wave solution to the second-order wave equation.

2. The Kirchhoff diffraction integral

The boundary conditions for Kirchhoff's diffraction equation demand that both the value (Dirichlet boundary condition) of an electromagnetic wave and its normal derivative (Neumann boundary condition) are continuous simultaneously at any surface in ordinary space as light propagates from a point (P_o) in one volume (V_a) to a point (P_2) in another volume (V_b) though a point (P_1) shared by the surfaces ($S_a \cap S_b$) that border both volumes (Fig. 1). The two volumes are separated by an opaque surface with an aperture of arbitrary shape that defines the shared surface. Kirchhoff's diffraction integral sums all the continuous optical paths originating at P_o and arriving at P_2 through the shared surface while taking the phase of each path into consideration. I will describe each optical path of the binary photons propagating from the source to the image point using the twice-differentiable scalar functions U and G, and their normal derivatives, which are also differentiable. The twice differentiability satisfies the second-order wave equation. The condition for continuity between two volumes (V_a and V_b) that share a surface requires that for some function X, $X_a = X_b$ and for another function Y, $\frac{\partial Y_a}{\partial n} = \frac{\partial Y_b}{\partial n}$. The intensity of the light that propagates from P_o to P_2 in the diffraction plane is obtained by multiplying the Kirchhoff diffraction integral by its complex conjugate.

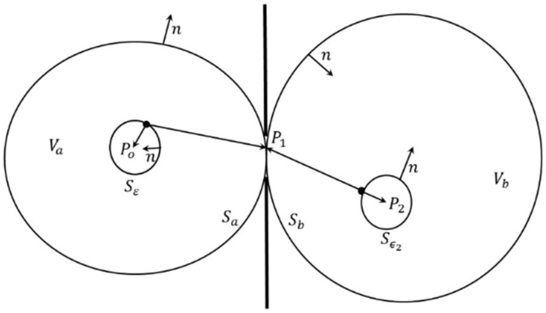

Fig. 1: The propagation of light from P_o to P_2 by way of a point P_1 in the opening of the aperture, where the arbitrary surfaces S_a and S_b intersect. The normals are outward with respect to V_a and inward with respect to V_b. S_ϵ is a spherical surface that surrounds the source point of light and S_{ϵ_2} is the spherical surface that surrounds the image point in the diffraction plane. In the hole in the aperture, $X_a = X_b$ and $\frac{\partial Y_a}{\partial n} = \frac{\partial Y_b}{\partial n}$ and in the opaque regions of the aperture, $X_a = X_b = 0$ and $\frac{\partial Y_a}{\partial n} = \frac{\partial Y_b}{\partial n} = 0$.

With respect to Green [1], the magnetic and electric properties of light will be treated in terms of their magnetic and electric potentials (in V)[1]. The magnetic potential $\left(\frac{cB}{k}\right)$ is obtained by multiplying the magnetic flux density vector (**B** in Vs/m^2) by the radius of the binary photon $\left(\text{radius} = \frac{1}{k} = \frac{\lambda}{2\pi} \text{ in m}\right)$ and the speed of light (c). Likewise, the electric potential $\left(\frac{iE}{k}\right)$ is obtained by multiplying the electric field (**E**, in V/m) by the radius of the binary photon and the imaginary number $i = \sqrt{-1}$, which indicates that the electric potential at a point is orthogonal to the magnetic potential at that point. The electric potential at a point is orthogonal to the magnetic potential in a manner consistent with Faraday's law, the Ampere-Maxwell law and the right hand rule. The ratio of the magnetic potential to the electric potential of the binary photon throughout a period is equal to the speed of light. The relationships between the magnetic and the electrical properties at a point (P) are given in Table 1.

The time-independent functions $U(P)$ and $G(P)$ at any given point (P) that are based on the magnetic potential $\left(\frac{c\boldsymbol{B}}{k}\right)$ and the electric potential $\left(\frac{i\boldsymbol{E}}{k}\right)$ of the binary photon are defined like so:

$$U(P) = \sqrt{\frac{\varepsilon_o}{\mu_o}\left(\sqrt{2}\,\frac{c\boldsymbol{B}_o}{k}\right)}\,\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\boldsymbol{r}} \quad (2)$$

$$G(P) = \sqrt{\sqrt{\frac{\mu_o}{\varepsilon_o}}\left(\sqrt{2}\,\frac{i\boldsymbol{E}_o}{k}\right)}\,\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\boldsymbol{r}} \quad (3)$$

Where, ε_o is the electric permittivity of the vacuum and μ_o is the magnetic permeability of the vacuum. While $\boldsymbol{B}$ and $\boldsymbol{E}$ are vectors that represent the magnetic flux density in the *xz* plane and the electric field in the *yz* plane [34], they will be treated as scalars, which is true for axially-symmetrical un-polarized light. This analysis can be extended for the case of polarized light where $U(P)$ and $G(P)$ would be vectors.

The normal derivative of Eqns. (2) and (3) are:

$$\frac{\partial U}{\partial n}(P) = \frac{\left(-\frac{e^{ikr}}{r^2} + \frac{ie^{ikr}}{r}\right)\sqrt{\sqrt{2}\frac{c\boldsymbol{B}_o}{k}\left(\frac{\varepsilon_o}{\mu_o}\right)}}{2\sqrt{\frac{e^{ikr}}{r}}}\cos(\boldsymbol{n}\cdot\boldsymbol{r})$$

$$= \frac{\left(ik - \frac{1}{r}\right)\sqrt{\left(\frac{\varepsilon_o}{\mu_o}\right)\left(\sqrt{2}\frac{c\boldsymbol{B}_o}{k}\right)}\sqrt{\frac{e^{ikr}}{r}}}{2}\cos(\boldsymbol{n}\cdot\boldsymbol{r}) \quad (4)$$

and

$$\frac{\partial G}{\partial n}(P) = \frac{\left(-\frac{e^{ikr}}{kr^2} + \frac{ie^{ikr}}{r}\right)\sqrt{\sqrt{2}\frac{i\boldsymbol{E}_o}{k}\sqrt{\frac{\mu_o}{\varepsilon_o}}}}{2\sqrt{\frac{e^{ikr}}{kr}}}\cos(\boldsymbol{n}\cdot\boldsymbol{r})$$

$$= \frac{\left(ik - \frac{1}{r}\right)\sqrt{\sqrt{\frac{\mu_o}{\varepsilon_o}}\left(\sqrt{2}\frac{i\boldsymbol{E}_o}{k}\right)}\sqrt{\frac{e^{ikr}}{r}}}{2}\cos(\boldsymbol{n}\cdot\boldsymbol{r}) \quad (5)$$

Since the ratio of the magnetic potential to the electric potential of the binary photon throughout a period is equal to the speed of light, $c\boldsymbol{B}_o = i\boldsymbol{E}_o$, and consistent with Faraday's law and the Ampere-Maxwell law, Eqns. (4) and (5) become:

$$\frac{\partial U}{\partial n}(P) = \frac{\left(-\frac{e^{ikr}}{r^2} + \frac{ie^{ikr}}{r}\right)\sqrt{\sqrt{2}\frac{i\boldsymbol{E}_o}{k}\left(\frac{\varepsilon_o}{\mu_o}\right)}}{2\sqrt{\frac{e^{ikr}}{r}}}\cos(\boldsymbol{n}\cdot\boldsymbol{r})$$

$$= \frac{\left(ik - \frac{1}{r}\right)\sqrt{\left(\frac{\varepsilon_o}{\mu_o}\right)\left(\sqrt{2}\frac{i\boldsymbol{E}_o}{k}\right)}\sqrt{\frac{e^{ikr}}{r}}}{2}\cos(\boldsymbol{n}\cdot\boldsymbol{r}) \quad (6)$$

and

$$\frac{\partial G}{\partial n}(P) = \frac{\left(-\frac{e^{ikr}}{kr^2} + \frac{ie^{ikr}}{r}\right)\sqrt{\sqrt{2}\frac{c\boldsymbol{B}_o}{k}\sqrt{\frac{\mu_o}{\varepsilon_o}}}}{2\sqrt{\frac{e^{ikr}}{kr}}}\cos(\boldsymbol{n}\cdot\boldsymbol{r})$$

[1] The fields can be obtained by taking the negative gradient of the potentials, and the potential energy can be obtained from the potential by multiplying the potential by the charge.

$$= \frac{\left(ik-\frac{1}{r}\right)\sqrt{\sqrt{\frac{\mu_o}{\varepsilon_o}}\left(\sqrt{2}\frac{c\boldsymbol{B_o}}{k}\right)}\sqrt{\frac{e^{ikr}}{r}}}{2}\cos(\boldsymbol{n}\cdot\boldsymbol{r}) \quad (7)$$

Taking the product of Eqns. (2) and (7) and Eqns. (3) and (6), we get:

$$U\frac{\partial G}{\partial \boldsymbol{n}}(P) = \sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}\frac{e^{ikr}}{\boldsymbol{r}}\left(ik-\frac{1}{r}\right)\cos(\boldsymbol{n}\cdot\boldsymbol{r}) \quad (8a)$$

$$G\frac{\partial U}{\partial \boldsymbol{n}}(P) = \sqrt{\frac{c\varepsilon_o i\boldsymbol{E_o}\cdot i\boldsymbol{E_o}}{2k^2}}\frac{e^{ikr}}{\boldsymbol{r}}\left(ik-\frac{1}{r}\right)\cos(\boldsymbol{n}\cdot\boldsymbol{r}) \quad (8b)$$

Green's theorem requires that

$$U\frac{\partial G}{\partial \boldsymbol{n}}(P) - G\frac{\partial U}{\partial \boldsymbol{n}}(P) = 0 \quad (9).$$

We can show that this requirement is met by substituting Eqns. (8a) and (8b) into Eqn. (9):

$$\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}\frac{e^{ikr}}{\boldsymbol{r}}\left(ik-\frac{1}{r}\right)\cos(\boldsymbol{n}\cdot\boldsymbol{r}) -$$

$$\sqrt{\frac{c\varepsilon_o i\boldsymbol{E_o}\cdot i\boldsymbol{E_o}}{2k^2}}\frac{e^{ikr}}{\boldsymbol{r}}\left(ik-\frac{1}{r}\right)\cos(\boldsymbol{n}\cdot\boldsymbol{r}) = 0 \quad (10)$$

Since $\varepsilon_o\mu_o = \frac{1}{c^2}$, after rearranging, we get:

$$\left[\sqrt{c\boldsymbol{B}_o\cdot c\boldsymbol{B}_o} - \sqrt{i\boldsymbol{E}_o\cdot i\boldsymbol{E}_o}\,\right]\frac{\sqrt{\varepsilon_o}}{2\sqrt{\mu_o}k^2}$$

$$\frac{e^{ikr}}{\boldsymbol{r}}\left(ik-\frac{1}{r}\right)\cos(\boldsymbol{n}\cdot\boldsymbol{r}) = 0 \quad (11)$$

From Faraday's law and the Ampere-Maxwell law, it follows that $c\boldsymbol{B_o} = i\boldsymbol{E_o}$. Therefore we get:

$$\left[\sqrt{i\boldsymbol{E}_o\cdot i\boldsymbol{E}_o} - \sqrt{i\boldsymbol{E}_o\cdot i\boldsymbol{E}_o}\,\right]\frac{\sqrt{\varepsilon_o}}{2\sqrt{\mu_o}k^2}$$

$$\frac{e^{ikr}}{\boldsymbol{r}}\left(ik-\frac{1}{r}\right)\cos(\boldsymbol{n}\cdot\boldsymbol{r}) = 0 \quad (12)$$

indicating that the functions U and G and their first spatial derivatives satisfy the requirements for Green's theorem.

According to the binary photon theory, the Dirichlet and Neumann boundary conditions are instantaneously and simultaneously satisfied by the following functional pairs: $U(P_a) = U(P_b)$, $\frac{\partial G}{\partial \boldsymbol{n}}(P_a) = \frac{\partial G}{\partial \boldsymbol{n}}(P_b)$, $G(P_a) = G(P_b)$, $\frac{\partial U}{\partial n}(P_a) = \frac{\partial U}{\partial n}(P_b)$; $U\frac{\partial G}{\partial \boldsymbol{n}}(P_a) = U\frac{\partial G}{\partial \boldsymbol{n}}(P_b)$ and $G\frac{\partial U}{\partial n}(P_a) = G\frac{\partial U}{\partial n}(P_b)$. Moreover, the fields do not vanish identically at an arbitrary surface as they would [27] according to Maxwell's [26] description of light as an electromagnetic wave because in the binary photon, the electric field in the *yz* plane, which is a quadrature out-of-phase with the magnetic field in the *xz* plane, regenerates the magnetic field according to the Ampere-Maxwell law. Likewise, the magnetic field in the *xz* plane, which is a quadrature out-of-phase with the electric field in the *yz* plane, regenerates the electric field according to Faraday's law. U and G, which are twice differentiable, represent light with a wavenumber k, and obey the Helmholtz equation:

$$(\nabla^2 + k^2)U = 0 \quad (13a)$$

$$(\nabla^2 + k^2)G = 0 \quad (13b)$$

Substituting Eqns. (13a) and (13b) into Eqn. (1), the RHS becomes:

$$\iiint\left(U(-k^2G) - G(-k^2U)\right)dV =$$

$$\iiint k^2(GU - UG)\,dV \quad (14)$$

And since $GU - UG$ on the RHS of Eqn. (14) vanishes, Eqn. (1) becomes:

$$\iiint_V (U\nabla^2 G - G\nabla^2 U)\,dV =$$

$$\iint_S \left(U\frac{\partial G}{\partial n} - G\frac{\partial U}{\partial n}\right)dS = 0 \quad (15)$$

Spherical waves are usually described as having spherical surfaces with constant phase. Here I describe spherical waves as a surface that subtends the propagation of binary photons along radial rays where the spherical surface simultaneously allows for the constant phase relationship between the electric and magnetic components that are quadrature out-of-phase. The binary photons, with their orthogonal and alternating electric and magnetic compenents propagate from the luminous point source (P_o) to point (P_1) on arbitrary surface (S_1). Since the point source (P_o) represents a discontinuity that is incompatible with the assumptions of Green's theorem, we have to apply limits during the integration of the surface surrounding the point source to remove P_o from the arbitrary volume (V_1) in the object space and describe S_1 as being an arbitrary surface that is consistent with the assumptions of Green's theorem where $\iint_S \left(U\frac{\partial G}{\partial \boldsymbol{n}} - G\frac{\partial U}{\partial \boldsymbol{n}}\right)dS$ vanishes (Fig. 2). To do this, we consider two surfaces S_1 and S_ϵ that surround a volume (V_1). The normals to the two surfaces point away from V_1. S_ϵ is a spherical surface with radius ϵ that excludes the source in volume ($V_\epsilon = \frac{4\pi}{3}\epsilon^3$) from V_1.

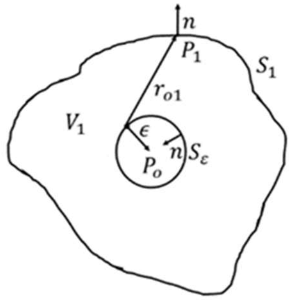

Fig. 2: Definitions of the volume, surfaces, rays and vectors in the object space.

As a result of the oppositely directed normals surrounding V_1, it follows from Eqn. (15) that the sum of the two surface integrals that make up $\iint_S \left(U\frac{\partial G}{\partial n} - G\frac{\partial U}{\partial n}\right) dS$ also vanishes:

$$\iint_{S_\epsilon}\left(U_\epsilon\frac{\partial G_\epsilon}{\partial n} - G_\epsilon\frac{\partial U_\epsilon}{\partial n}\right)dS_\epsilon + \iint_{S_1}\left(U_{S_1}\frac{\partial G_{S_1}}{\partial n} - G_{S_1}\frac{\partial U_{S_1}}{\partial n}\right)dS_1 = 0 \quad (16)$$

Consequently,

$$\iint_{S_1}\left(U_{S_1}\frac{\partial G_{S_1}}{\partial n} - G_{S_1}\frac{\partial U_{S_1}}{\partial n}\right)dS_1 = -\iint_{S_\epsilon}\left(U_\epsilon\frac{\partial G_\epsilon}{\partial \boldsymbol{n}} - G_\epsilon\frac{\partial U_\epsilon}{\partial \boldsymbol{n}}\right)dS_\epsilon \quad (17)$$

Substitute Eqns. (8a) and (8b) in Eqn. (17) and simplify to get:

$$\iint_{S_1}\left(U_{S_1}\frac{\partial G_{S_1}}{\partial n} - G_{S_1}\frac{\partial U_{S_1}}{\partial n}\right)dS_1 = -\iint_{S_\epsilon}\left(ik - \frac{1}{\epsilon}\right)\frac{e^{ik\epsilon}}{\epsilon}\cos(\boldsymbol{n}\cdot\boldsymbol{\epsilon})\left[\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right]dS_\epsilon \quad (18)$$

Since ϵ is a radius of S_ϵ, then $dS_\epsilon = \epsilon^2 d\Omega$, and Eqn. (18) becomes:

$$\iint_{S_1}\left(U_{S_1}\frac{\partial G_{S_1}}{\partial n} - G_{S_1}\frac{\partial U_{S_1}}{\partial n}\right)dS_1 =$$

$$\iint_\Omega \left(ik - \frac{1}{\epsilon}\right)\frac{e^{ik\epsilon}}{\epsilon}\left[\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right]\epsilon^2 d\Omega \quad (19)$$

Distribute ϵ^2 and take the limit as $\epsilon \to 0$:

$$\lim_{\epsilon\to 0}\iint_\Omega \left(ik\epsilon - \frac{\epsilon}{\epsilon}\right)e^{ik\epsilon}\left[\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right]d\Omega = -4\pi\left[\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right] \quad (20)$$

Let $\left[\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right] = A_o$, which is the complex amplitude in $\left(\frac{J}{s}\right)^{\frac{1}{2}}$ of the source point (P_o). Now let r_{o1} represent the distance between P_o and P_1 on S_1:

$$\iint_{S_1}\left(ik - \frac{1}{r_{o1}}\right)\frac{e^{ikr_{o1}}}{r_{o1}}\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})\left[\sqrt{\frac{c\boldsymbol{B_1}\cdot\boldsymbol{B_1}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_1}\cdot\boldsymbol{E_1}}{2k^2}}\right]dS_1 = -4\pi A_o \quad (21)$$

After rearranging, we get:

$$A_o = -\frac{1}{4\pi}\iint_{S_1}\left(ik - \frac{1}{r_{o1}}\right)\frac{e^{ik\ _{o1}}}{r_{o1}}\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})\left[\sqrt{\frac{c\boldsymbol{B_1}\cdot\boldsymbol{B_1}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_1}\cdot\boldsymbol{E_1}}{2k^2}}\right]dS_1 \quad (22)$$

If $k \gg \frac{1}{r_{o1}}$, Eqn. (22) becomes:

$$A_o = -\frac{1}{4\pi}\iint_{S_1}\frac{e^{ik\ _{o1}}}{r_{o1}}ik\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})\left[\sqrt{\frac{c\boldsymbol{B_1}\cdot\boldsymbol{B_1}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_1}\cdot\boldsymbol{E_1}}{2k^2}}\right]dS_1 \quad (23)$$

After taking ik, which is assumed to be constant, out of the integral, we get:

$$A_o = -\frac{ik}{4\pi}\iint_{S_1}\frac{e^{ikr_{o1}}}{r_{o1}}\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})\left[\sqrt{\frac{c\boldsymbol{B_1}\cdot\boldsymbol{B_1}}{2\mu_o k^2}} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_1}\cdot\boldsymbol{E_1}}{2k^2}}\right]dS_1 \quad (24)$$

Now consider the surfaces that surround a volume (V_2) in the image space that excludes the image point (P_2) and then let the surface nearest the image point vanish (Fig. 3). The surface normals point towards V_2. S_{1a} intersects with S_1 (Fig. 2) where there is a hole in such a way that at P_1 both $U\frac{\partial G}{\partial n}$ and $G\frac{\partial U}{\partial n}$ are continuous.

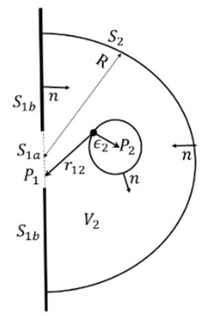

Fig. 3: Definitions of the volume, surfaces, rays and vectors in the image space.

Some of the light incident on S_1 can be excluded from the image space by a perfectly conducting or perfectly black aperture support represented by S_{1b}. Both $U\frac{\partial G}{\partial n}$ and $G\frac{\partial U}{\partial n}$ vanish at every point on S_{1b}. S_2 represents the surface which along with S_{1a} and S_{1b} surrounds the volume in the image space containing the image point (P_2). Since the image point (P_2) represents a discontinuity that is incompatible with the assumptions of Green's theorem, we have to remove P_2 from volume (V_2) just as we removed P_o from volume (V_1), and describe S_2, S_{1a} and S_{1b} as being a surface that is consistent with the assumptions of Green's theorem where $\iint\left(U\frac{\partial G}{\partial n}-G\frac{\partial U}{\partial n}\right)dS$ vanishes. The complete surface integral surrounding V_2 is given by:

$$\iint_S\left(U\frac{\partial G}{\partial n}-G\frac{\partial U}{\partial n}\right)dS =$$

$$\iint_{S_{\epsilon_2}}\left(U_{\epsilon_2}\frac{\partial G_{\epsilon_2}}{\partial n}-G_{\epsilon_2}\frac{\partial U_{\epsilon_2}}{\partial n}\right)dS_{\epsilon_2} +$$

$$\iint_{S_{1a}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1a} +$$

$$\iint_{S_{1b}}\left(U_{1b}\frac{\partial G_{S_{1b}}}{\partial n}-G_{S_{1b}}\frac{\partial U_{1b}}{\partial n}\right)dS_{1b} +$$

$$\iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2 = 0 \qquad (25)$$

Thus we can determine the value of the integral of the spherical surface that surrounds the image point:

$$-\iint_{S_{\epsilon_2}}\left(U_{\epsilon_2}\frac{\partial G_{\epsilon_2}}{\partial n}-G_{\epsilon_2}\frac{\partial U_{\epsilon_2}}{\partial n}\right)dS_{\epsilon_2} = \iint_{S_{1a}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1a} + \iint_{S_{1b}}\left(U_{1b}\frac{\partial G_{S_{1b}}}{\partial n}-G_{S_{1b}}\frac{\partial U_{1b}}{\partial n}\right)dS_{1b} + \iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2 \qquad (26)$$

Substitute Eqns. (8a) and (8b) into Eqn. (26) to get:

$$\iint_{S_{1a}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1a} + \iint_{S_{1b}}\left(U_{1b}\frac{\partial G_{S_{1b}}}{\partial n}-G_{S_{1b}}\frac{\partial U_{1b}}{\partial n}\right)dS_{1b} + \iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2 = -\iint_{S_{\epsilon_2}}\left(ik-\frac{1}{\epsilon_2}\right)\frac{e^{ik\epsilon_2}}{\epsilon_2}\cos(\boldsymbol{n}\cdot\boldsymbol{\epsilon_2})\left[\sqrt{\frac{c\boldsymbol{B_2}\cdot\boldsymbol{B_2}}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_2}\cdot\boldsymbol{E_2}}{2k^2}}\right]dS_{\epsilon_2} \qquad (27)$$

Since ϵ_2 is a radius of S_{ϵ_2}, then $dS_{\epsilon_2} = \epsilon_2^2 d\Omega$, and Eqn. (27) becomes:

$$\iint_{S_{1a}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1a} + \iint_{S_{1b}}\left(U_{1b}\frac{\partial G_{S_{1b}}}{\partial n}-G_{S_{1b}}\frac{\partial U_{1b}}{\partial n}\right)dS_{1b} + \iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2 = -\iint_{\Omega}\left(ik-\frac{1}{\epsilon_2}\right)\frac{e^{ik\ 2}}{\epsilon_2}\cos(\boldsymbol{n}\cdot\boldsymbol{\epsilon_2})\left[\sqrt{\frac{c\boldsymbol{B_2}\cdot\boldsymbol{B_2}}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_2}\cdot\boldsymbol{E_2}}{2k^2}}\right]\epsilon_2^2 d\epsilon_2 \qquad (28)$$

Distribute ϵ_2^2 and take the limit as $\epsilon_2 \to 0$:

$$\lim_{\epsilon_2\to 0}\iint_{\Omega}\left(ik-\frac{1}{\epsilon_2}\right)\frac{e^{ik\epsilon_2}}{\epsilon_2}\left[\sqrt{\frac{c\boldsymbol{B_2}\cdot\boldsymbol{B_2}}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_2}\cdot\boldsymbol{E_2}}{2k^2}}\right]\epsilon_2^2 d\Omega = -4\pi\left[\sqrt{\frac{c\boldsymbol{B_2}\cdot\boldsymbol{B_2}}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_2}\cdot\boldsymbol{E_2}}{2k^2}}\right] \qquad (29)$$

Let $\left[\sqrt{\frac{c\boldsymbol{B_2}\cdot\boldsymbol{B_2}}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_2}\cdot\boldsymbol{E_2}}{2k^2}}\right] = A_2$, which is the complex amplitude in $\left(\frac{J}{s}\right)^{\frac{1}{2}}$ of the image point (P_2), and Eqn. (28) becomes:

$$\iint_{S_{1a}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1a} + \iint_{S_{1b}}\left(U_{1b}\frac{\partial G_{S_{1b}}}{\partial n}-G_{S_{1b}}\frac{\partial U_{1b}}{\partial n}\right)dS_{1b} + \iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2 = -4\pi A_2 \qquad (30)$$

Since Green's theorem demands that $U_{1b}\frac{\partial G_{S_{1b}}}{\partial n}-G_{S_{1b}}\frac{\partial U_{1b}}{\partial n}$ vanishes on S_{1b}, $\iint_{S_{1b}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1b} = 0$ and Eqn. (30) becomes:

$$\iint_{S_{1a}}\left(U_{S_{1a}}\frac{\partial G_{S_{1a}}}{\partial n}-G_{S_{1a}}\frac{\partial U_{S_{1a}}}{\partial n}\right)dS_{1a} + \iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2 = -4\pi A_2 \qquad (31)$$

If R is considered to be so large that as $R \to \infty$, any light coming from S_2 would take so long to reach the image we can consider the contribution of

$\iint_{S_2}\left(U_{S_2}\frac{\partial G_{S_2}}{\partial n}-G_{S_2}\frac{\partial U_{S_2}}{\partial n}\right)dS_2$ to the image point to vanish. This is known as the Sommerfeld assumption. After letting r_{12} represent the distance between P_1 and P_2 and substituting Eqns. (8a) and (8b) into Eqn. (31), we get:

$$A_2 = -\frac{i}{4\pi}\iint_{S_{1a}}\left(ik-\frac{1}{r_{12}}\right)\frac{e^{ikr_{12}}}{r_{12}}\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\left[\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}\right]dS_{1a} \quad (32)$$

If $k \gg \frac{1}{r_{12}}$, Eqn. (32) becomes:

$$A_2 = -\frac{1}{4\pi}\iint_{S_{1a}}\frac{e^{ikr_{12}}}{r_{12}}ik\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\left[\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}\right]dS_{1a} \quad (33)$$

Since ik is a constant, we can move it outside the integral to get:

$$A_2 = -\frac{ik}{4\pi}\iint_{S_{1a}}\frac{e^{ik\ \ 12}}{r_{12}}\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\left[\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}\right]dS_{1a} \quad (34)$$

The complex amplitude $\left[\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}\right]$ of the electromagnetic power that passes S_{1a} is equal to the amplitude at S_1. Taking the sign of the cosines into consideration (Fig. 4), we can add Eqn. (34) ($A_2 = -\frac{ik}{4\pi}\iint_{S_{1a}}\frac{e^{ikr_{12}}}{r_{12}}\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\left[\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}\right]dS_{1a}$) to Eqn. (24) ($A_o = -\frac{ik}{4\pi}\iint_{S_1}\frac{e^{ikr_{o1}}}{r_{o1}}\cos(\boldsymbol{n}\cdot\boldsymbol{r_{o1}})\left[\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}+i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}\right]dS_1$) and square it to get the intensity (in J m^{-2} s^{-1}) of the light composed of binary photons diverging from the source in the object space, passing through the aperture and converging on the image point in the image space.

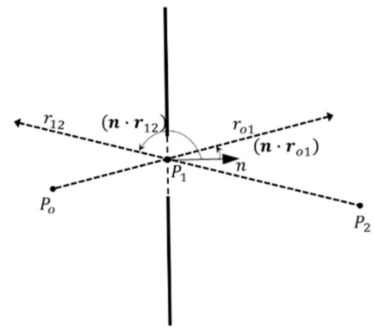

Fig. 4: The definitions of rays and vectors at the opening of the aperture where the arbitrary surfaces surrounding the object space and the image space intersect.

After cancelling signs, we get:

$$I_2 = \left|\frac{ik\ \ _o}{4\pi}\iint_{S_{ia}}\frac{e^{ik(r_{o1}+r_{12})}}{r_{o1}r_{12}}\left[\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})\right]-\left[\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\right]\ dS_{ia}\right|^2 \quad (35)$$

The squared modulus means to multiply the term inside the line brackets by its complex conjugate. The exponential in Eqn. (35) can also be written in terms of cosines and sines where one wave represents the magnetic component of light and the other represents the electrical component:

$$I_2 = \left|\frac{ik\ \ _o}{4\pi}\iint_{S_{ia}}\frac{\cos k(r_{o1}+r_{12})+i\sin\ \ (r_{o1}+r_{12})}{r_{o1}r_{12}}\left[\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})\right]-\left[\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\right]\ dS_{ia}\right|^2 \quad (36)$$

Thus the Kirchhoff diffraction integral can be derived based on the binary photon, with its alternating magnetic and electric components that simultaneously satisfy the Dirichlet and Newmann boundary conditions. As long as the wavelength is much smaller than the radius of the aperture and the radius of the aperture is much smaller than r_{o1} and r_{12}, we can simplify the Kirchhoff diffraction equation based on the binary photon to account for Fresnel and Fraunhofer diffraction. When the assumptions are met, $r_{o1}r_{12}$ can be replaced by $r'_{o1}r'_{12}$ and $\frac{[\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})-\mathrm{co}\ \ (\boldsymbol{n}\cdot\boldsymbol{r}_{12})]}{r'_{o1}r'_{12}}$ will not vary significantly over the aperture (Fig. 5). Consequently, $\frac{[\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{o1})-\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})]}{r'_{o1}r'_{12}}$ can be moved outside the integral and Eqn. (35) becomes:

$$I_2 = \left|\frac{ikA_o\left[\cos(\boldsymbol{n}\cdot\boldsymbol{r_{o1}})-\cos(\boldsymbol{n}\cdot\boldsymbol{r}_{12})\right]}{4\pi r'_{o1}r'_{12}}\iint_{S_{ia}}e^{ik(r_{o1}+r_{12})}\ dS_{ia}\right|^2 \quad (37)$$

Moreover, if the angles that rays P_oP_1 and P_2P_1 make with ray P_oP_2 are small, then v can be replaced by $2\cos\beta$, where β is the angle between ray P_2P_1 and the normal ($\boldsymbol{n}$) to the aperture (Fig. 5), which gives:

$$I_2 = \left|\frac{2ik\ _o\cos\beta}{4\pi r'_{o1}r'_{12}}\iint_{S_{ia}} e^{ik(r_{o1}+r_{12})}\ dS_{ia}\right|^2 \quad (38)$$

leaving only the phase inside the integral.

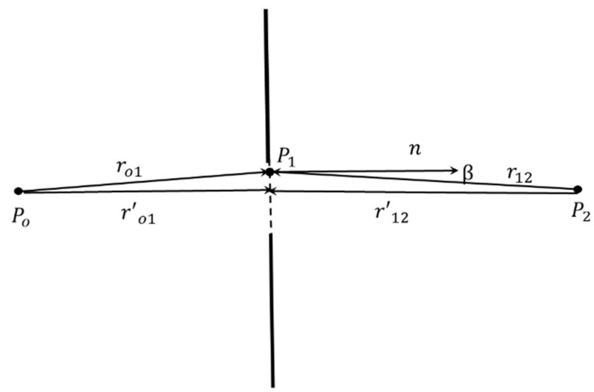

Fig. 5: β is the angle between ray P_2P_1 and the normal ($\boldsymbol{n}$). It is used for the Fresnel and Fraunhofer approximation when the angles that rays P_oP_1 and P_2P_1 make with ray P_oP_2 are small.

3. The two-dimensional diffraction pattern for apertures of various forms

The Kirchhoff diffraction equation derived on the basis of the binary photon can be simplified to approximate the Fresnel and Fraunhofer diffraction patterns formed by various apertures [35-39]. Here we will use three Cartesian reference systems—one that contains the source plane that includes P_o, with coordinates $(x_o, y_o, 2\zeta)$; one that contains the aperture plane that includes P_1, with coordinates (ξ, η, ζ); and one that contains the image plane that contains P_2, with coordinates (x_2, y_2, z_2). The three coordinate systems share the same z axis, where $z = 0$ at the image plane; $z = \zeta$ at the aperture plane; and $z = 2\zeta$ at the source plane. The common origin of the three coordinate systems is (0,0,0) in the image plane. The light propagates in the $+z$ direction. According to the Huygens-Fresnel principle, we will integrate the contribution of each point (ξ, η, ζ) on the aperture plane as if were a point source of light emitting binary photons in a radially-symmetrical manner so that they reach each point on the image plane, including P_2.

Define

$$r_{o1}^2 = (x_o - \xi)^2 + (y_o - \eta)^2 + (2\zeta - \zeta)^2 \quad (39a)$$

$$r_{12}^2 = (x_2 - \xi)^2 + (y_2 - \eta)^2 + (z_2 - \zeta)^2 \quad (39b)$$

$${r'_{o1}}^2 = (x_o - 0)^2 + (y_o - 0)^2 + (2\zeta - \zeta)^2 \quad (39c)$$

$${r'_{12}}^2 = (x_2 - 0)^2 + (y_2 - 0)^2 + (2\zeta - \zeta)^2 \quad (39d)$$

which after simplifying gives:

$$r_{o1}^2 = {r'_{o1}}^2 - 2(x_o\xi + y_o\eta) + \xi^2 + \eta^2 \quad (40a)$$

$$r_{12}^2 = {r'_{12}}^2 - 2(x_2\xi + y_2\eta) + \xi^2 + \eta^2 \quad (40b)$$

Since the dimensions of the aperture are small compared to $r'_{o1}$and r'_{12}, we can expand r_{o1} as a power series in $\frac{\xi}{r'_{o1}}$ and $\frac{\eta}{r'_{o1}}$ and expand r_{12} as power series in $\frac{\xi}{r'_{12}}$ and $\frac{\eta}{r'_{12}}$:

$$r_{o1} \approx r'_{o1} - \frac{x_o\xi + y_o\eta}{r'_{o1}} + \frac{\xi^2+\eta^2}{2r'_{o1}} - \frac{(x_o\xi + y_o\eta)^2}{2{r'_{o1}}^3} - \cdots \quad (41a)$$

$$r_{12} \approx r'_{12} - \frac{x_2\xi + y_2\eta}{r'_{12}} + \frac{\xi^2+\eta^2}{2r'_{12}} - \frac{(x_2\xi + y_2\eta)^2}{2{r'_{12}}^3} - \cdots \quad (41b)$$

Substitute Eqns. (41a) and (41b) into Eqn. (38) to get:

$$I_2 = \left|\frac{2ikA_o\cos\beta}{4\pi r'_{o1}r'_{12}}\iint_{S_{ia}} e^{ikf(\xi,\eta)}\ d\xi d\eta\right|^2 \quad (42)$$

where $(r_{o1} + r_{12})$ is now an explicit function $f()$ of ξ and η.

$$f(\xi,\eta) = -\frac{x_o\xi + y_o\eta}{r'_{o1}} - \frac{x_2\xi + y_2\eta}{r'_{12}} + \frac{\xi^2+\eta^2}{2r'_{o1}} + \frac{\xi^2+\eta^2}{2r'_{12}} - \frac{(x_o\xi + y_o\eta)^2}{2{r'_{o1}}^3} - \frac{(x_2\xi + y_2\eta)^2}{2{r'_{12}}^3} - \cdots \quad (43)$$

Let $l_o = -\frac{x_o}{r'_{o1}}$, $m_o = -\frac{y_o}{r'_{o1}}$, $l = -\frac{x_2}{r'_{12}}$, and $m = -\frac{y_2}{r'_{12}}$ so that $f(\xi,\eta)$ can be written in terms of the direction cosines as:

$$f(\xi,\eta) = (l_o - l)\xi + (m_o - m)\eta + \frac{1}{2}\left\{\left(\frac{1}{r'_{o1}} + \frac{1}{r'_{12}}\right)(\xi^2 + \eta^2) - \frac{(l_o\xi + m_o\eta)^2}{r'_{o1}} - \frac{(l\xi\quad m\eta)^2}{r'_{12}}\right\}\ldots \quad (44)$$

Inserting Eqn. (44) into Eqn. (42), we get:

$$I_2 = \left| \frac{2ikA_o\cos\beta}{4\pi r'_{01}r'_{12}} \iint_{S_{ia}} e^{ik\left((l_o-l)\xi+(m_o-m)\eta+\frac{1}{2}\left\{\left(\frac{1}{r'_{01}}+\frac{1}{r'_{12}}\right)(\xi^2+\eta^2)-\frac{(l_o\xi+m_o\eta)^2}{r'_{01}}-\frac{(l\xi+m\eta)^2}{r'_{12}}\right\}\right)} d\xi d\eta \right|^2 \quad (45)$$

Table 1. The Relationships between the Magnetic and the Electrical Properties at a Point (P).

	Magnetic Properties of Binary Photon	Units	Electric Properties of Binary Photon	Units
Potential	$\frac{cB}{k}(P)$	V	$\frac{iE}{k}(P)$	V
Function	$U(P)=\sqrt{\frac{\varepsilon_o}{\mu_o}\left(\sqrt{2}\frac{c\boldsymbol{B}}{k}\right)\frac{e^{ik\cdot r}}{\boldsymbol{r}}}$	$\frac{A}{\sqrt{Vm}}$	$G(P)=\sqrt{\sqrt{\frac{\mu_o}{\varepsilon_o}}\left(\sqrt{2}\frac{i\boldsymbol{E}}{k}\right)\frac{e^{ik\cdot r}}{\boldsymbol{r}}}$	$\frac{V}{\sqrt{Am}}$
Normal Derivative of Function	$\frac{\partial G}{\partial \boldsymbol{n}}(P)$	$\frac{V}{\sqrt{Am^3}}$	$\frac{\partial U}{\partial \boldsymbol{n}}(P)$	$\frac{A}{\sqrt{Vm^3}}$
	$U\frac{\partial G}{\partial \boldsymbol{n}}(P)$	$\sqrt{\frac{AV}{m^4}}$	$G\frac{\partial U}{\partial \boldsymbol{n}}(P)$	$\sqrt{\frac{AV}{m^4}}$
	$\left(U\frac{\partial G}{\partial \boldsymbol{n}}(P)\right)\left(U\frac{\partial G}{\partial \boldsymbol{n}}(P)\right)^*$	$\frac{J}{m^4s}$	$\left(G\frac{\partial U}{\partial \boldsymbol{n}}(P)\right)\left(G\frac{\partial U}{\partial \boldsymbol{n}}(P)\right)^*$	$\frac{J}{m^4s}$
Amplitude of Power	$\sqrt{\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}}$	$\sqrt{\frac{J}{s}}$	$\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}}$	$\sqrt{\frac{J}{s}}$
Power	$\frac{c\boldsymbol{B}_1\cdot\boldsymbol{B}_1}{2\mu_o k^2}$	$\frac{J}{s}$	$\frac{c\varepsilon_o\boldsymbol{E}_1\cdot\boldsymbol{E}_1}{2k^2}$	$\frac{J}{s}$

which is the integral based on the binary photon for Fresnel diffraction. When $\frac{k}{2}\left\{\left(\frac{1}{r'_{01}}+\frac{1}{r'_{12}}\right)(\xi^2+\eta^2)-\frac{(l_o\xi+m_o\eta)^2}{r'_{01}}-\frac{(l\xi+m\eta)^2}{r'_{12}}\right\} \ll 2\pi$, the quadratic and higher terms can be neglected in $f(\xi,\eta)$, the diffracted binary photons can be treated as binary photons propagating along parallel rays, and we get the integral based on the binary photon for Fraunhofer diffraction:

$$I_2 = \left|\frac{2ikA_o\cos}{4\pi r'_{01}r'_{12}}\iint_{S_{ia}} e^{ik((l_o-l)\xi+(m_o-m)\eta)}\ d\xi d\eta\right|^2 (46)$$

By treating $\frac{2ikA_o\cos\beta}{4\pi r'_{01}r'_{12}}$ as a constant, the diffracted binary photons can be treated as propagating along parallel rays. That is, the radially-symmetrical emission of binary photons from an object point and their propagation through an optical system can be represented by an integral whose integrand is a superposition of binary photons with various phases and directions of propagation. In order to solve the above integral equation, let $p = l_o - l$ and $q = m_o - m$, so that Eqn. (46) becomes:

$$I_2 = \left|\frac{2ikA_o\,\mathrm{co}}{4\pi r'_{01}r'_{12}}\iint_{S_{ia}} e^{ik(p\xi+q\ \ )}\ d\xi d\eta\right|^2 \quad (47)$$

Let $K = \frac{2ikA_o\cos}{4\pi r'_{01}r'_{12}}$ so Eqn. (47) becomes:

$$I_2 = \left|K\iint_{S_{ia}} e^{ik(p\xi+q\eta)}\ d\xi d\eta\right|^2 \quad (48)$$

Fresnel [40,41] discovered that the intensity of light in the diffraction plane depends on the size and shape of the aperture that separates the object space from the image space. We will consider two shapes: a rectangle and a circle. Firstly, assume the aperture is a rectangle with sides equal to 2a and 2b. Integrate Eqn. (48) by parts:

$$I_2 = \left|K \int_{-a}^{a}\int_{-b}^{b} e^{ik(p\xi+q\eta)}\ d\xi d\eta\right|^2 = \left|K \int_{-a}^{a} e^{ik(p\xi)} d\xi \int_{-b}^{b} e^{ik(q\eta)}\ d\eta\right|^2 \quad (49)$$

Since $\int_{-a}^{a} e^{ik(p\xi)} d\xi = \frac{e^{ikpa} - e^{-ikpa}}{ikp} = 2a\frac{\sin kpa}{kpa}$ and $\int_{-b}^{b} e^{ik(q\eta)}\ d\eta = \frac{e^{ikqb}+e^{-ikpb}}{ikq} = 2b\frac{\sin kqb}{kqb}$, then using the definition $\mathrm{sinc}(x) = \frac{\sin x}{x}$, Eqn. (49) becomes:

$$I_2 = \left[\left|K \int_{-a}^{a}\int_{-b}^{b} e^{ik(p\xi+q\eta)}\ d\xi d\eta\right|\right]^2 = |4abK\ \mathrm{sinc}(\ kpa)\ \mathrm{sinc}(\ kqb)|^2 \quad (50).$$

The sinc function is equivalent to a zeroth-order spherical Bessel function of the first kind and the Fourier transform of a rectangular function. When the aperture is a narrow slit, $b \gg \frac{1}{k}$, and $\mathrm{sinc}(\ kqb)$ varies rapidly with q such that the intensity falls off rapidly with q and the coherent line of light diffracted in the b direction acts like a point source of binary photons in the center of the slit. Thus for a slit, Eqn. (50) becomes

$$I_2 = |2aK\ \mathrm{sinc}(\ kpa)|^2 \quad (51).$$

This diffraction integral based on the binary photon can also be used in spectroscopy to characterize the positions of the spectral lines formed by a slit [42,43].

Now consider a circular aperture with a radius a, let $\xi = a\cos\theta$ and $\eta = a\sin\theta$. Then $f(\xi,\eta)$ in the aperture plane becomes:

$$f(\xi,\eta) = a(p\cos\theta + q\sin\theta) = a\sqrt{p^2+q^2}\left(\frac{p}{\sqrt{p^2+q^2}}\cos\theta + \frac{q}{\sqrt{p^2+q^2}}\sin\theta\right) \quad (52)$$

Let ρ represent the proportion of the distance along the radius of the aperture so that $\rho = \sqrt{p^2+q^2}$. Eqn. (52) becomes:

$$f(\xi,\eta) = a\rho\left(\frac{p}{\rho}\cos\theta + \frac{q}{\rho}\sin\theta\right) \quad (53)$$

Since in the diffraction plane, $\frac{p}{\rho} = \cos\varphi$ and $\frac{q}{\rho} = \sin\varphi$, then Eqn. (53) becomes

$$f(\xi,\eta) = a\rho(\cos\varphi\cos\theta + \sin\varphi\sin\theta) = a\rho\cos(\theta-\varphi) \quad (54)$$

and the diffraction integral based on the binary photon becomes:

$$I_2 = \left|K \int_{\rho=0}^{\rho=1}\int_{\theta=0}^{\theta=2\pi} e^{ika\rho\cos(\theta-\varphi)}\ \rho d\rho d\theta\right|^2 \quad (55)$$

Due to axial symmetry around the optic (z) axis, the solution of Eqn. (55) is independent of φ. Without any loss of generality, we let $\varphi = 0$ and Eqn. (55) becomes

$$I_2 = \left|K \int_{\rho=0}^{\rho=1}\int_{\theta=0}^{\theta=2\pi} e^{ika\rho\cos(\theta)}\ \rho d\rho d\theta\right|^2 \quad (56)$$

The integral cannot be reduced to fundamental rational functions such as sine, cosine, polynomial, logarithmic, or exponential functions. The integral can however be reduced to a Bessel function. The Bessel function can be defined by a series expansion around $x = 0$.

Using the definition of the Bessel function of the first kind of order zero: $J_o(u) = \frac{1}{2\pi}\int_{v=0}^{v=2\pi} e^{iu\cos v} dv$, we get:

$$\int_{\theta=0}^{\theta=2\pi} e^{ika\rho\cos\theta} d\theta = 2\pi J_o(ka\rho) \quad (57)$$

And Eqn. (56) becomes:

$$I_2 = \left|2\pi K \int_{\rho=0}^{\rho=1} J_o(ka\rho)\ \rho d\rho\right|^2 \quad (58)$$

Change the variable so that $ka\rho = x$, $\rho = \frac{x}{ka}$ and $d\rho = \frac{1}{ka}dx$, then Eqn. (58) becomes:

$$I_2 = \left|2\pi K \int_{\rho=0}^{\rho=1} J_o(ka\rho)\ \rho d\rho\right|^2 = \left|2\pi K \int_{\frac{x}{ka}=0}^{\frac{x}{ka}=1} J_o(x)\ \frac{x}{ka}\frac{dx}{ka}\right|^2 = \left|\frac{2\pi K}{(ka)^2}\int_{\frac{x}{ka}=0}^{\frac{x}{ka}=1} xJ_o(x)\ dx\right|^2 \quad (59)$$

Bessel functions of ascending orders are related by the recurrence relations where

$$\int_{x=0}^{x=1} x^{n+1}J_n(x)dx = x^{n+1}J_{n+1}(x) \quad (60)$$

Using Eqns. (59) and (60), letting $n = 0$ and integrating from $x = 0$ to $x = ka\rho$, Eqn. (58) becomes:

$$I_2 = \left|\frac{2\pi K}{(ka)^2}\int_{x=0}^{x=ka} xJ_o(x)dx\right|^2 = \left|\frac{2\pi K}{(ka)^2} ka\, J_1(x)\right|^2 = \left|2\pi K\frac{J_1(ka)}{ka}\right|^2 \quad (61)$$

Which is the standard diffraction equation for a circular aperture.

4. The three-dimensional diffraction pattern for a point source

According to geometrical optics, the image produced by a perfect lens is a point-by-point representation of the object [44]. The fact that a perfect image cannot be produced by an aberration-free lens led to the development of diffraction theory based on the wave theory of light [45,46].

Even with an aberration-free lens, the point-by-point relation between the object and the image depends on the photon being a mathematical point for if the photon had extension, a point in the object space would be represented by a three-dimensional volume in the image space that was related to the diameter and length of the photon. Every image shows latero-axial astigmatism in that a point in the image plane is not represented by a point in the object plane but by a prolate ellipsoid with respect to the optic axis and a circle with respect to the focal plane [47-49]. Historically, the prolate ellipsoid along the axis was modelled geometrically as an infinite thin slit [50] and formally by the sine function, which is equivalent to a zeroth-order spherical Bessel function of the first kind and a Fourier transform of a rectangular function; and the circle [51] in the focal plane was modelled geometrically as a circle and formally by a first-order Bessel function of the first kind [52- 55].[2] Based on these assumptions, Lommel [56, 57], Struve [58], and Schwarzschild [59] developed formulae to calculate the intensity far from the focal plane and this led to the standard treatment in describing the three-dimensional diffraction pattern produced by optical instruments [19,60-77].

I have modelled the binary photon as a propagating oscillator/rotator composed of two equal and opposite charges that produce a transverse electric field with a maximal diameter of $\frac{\lambda}{\pi}$ and a maximal axial length equal to λ in a single period [33,34]. Is it possible that the fact that a spherical point source gives rise to latero-axial astigmatism is a consequence of light not being composed of geometrical points but astigmatic binary photons? Is it possible that the lateral and axial diffraction patterns can both be formally modelled by the same first-order Bessel function of the first kind after latero-axial astigmatism of the binary photon is taken into consideration?

For the analysis, I assume Fraunhofer diffraction where the diameter ($2a$) of the aperture is much greater than the wavelength (λ). Consequently, the wave front at the aperture is considered to be a plane wave. The plane wave at the aperture can also be considered as the resultant of two sets of radially propagating binary photons travelling in opposite directions from P_o and P_2 to P_1 on the shared surface (Fig. 6). The diffraction pattern is also considered to be planar.

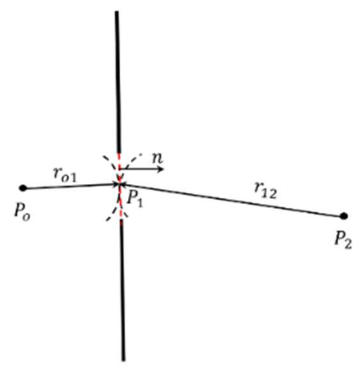

Fig. 6: The plane at the aperture can be considered as the resultant of two sets of radially propagating binary photons travelling in opposite directions from P_o and P_2 to P_1 on the shared surface. Likewise the plane wave can be considered to be the resultant of two diverging spherical waves that envelop the propagating binary photons travelling in opposite directions. The two sets of propagating binary photons meet simultaneously at any P_1 on the aperture plane and the aperture plane can be considered coincident with a plane wave (shown in red).

The three-dimensional diffraction pattern formed by a point source of light passing through a circular aperture can be determined according to Fig. 7 and the following definitions:

$$\xi = a\rho \sin\theta \tag{62a}$$

$$\eta = a\rho \cos\theta \tag{62b}$$

$$\zeta = \frac{a\rho}{\tan\sigma} \tag{62c}$$

$$x = R \sin\varphi \tag{62d}$$

$$y = R \cos\varphi \tag{62e}$$

$$z = \kappa \frac{R\cos}{\pi} \tag{62f}$$

$$R = \sqrt{x^2 + y^2 + z^2} \tag{62g}$$

[2] The spherical Bessel differential equation for functions of order m is $x^2\frac{\partial^2 A}{\partial x^2} + 2A\frac{dA}{dx} + (x^2 - m(m+1))x = 0$ and the Bessel differential equation for functions of order n is $x^2\frac{\partial^2 A}{\partial x^2} + 2A\frac{dA}{dx} + (x^2 - n^2)x = 0$ where A is an amplitude, x is a distance, m is an integer that serves as an index and n is a nonnegative integer that serves as the index. The solutions to these equations are a series of ascending powers of x.

where ρ is a proportion of the radius a of the aperture; κ is a proportion of the length R from the focal plane along the z axis; and π normalizes the axes of the Cartesian coordinate system relative to the asymmetry of the binary photon.

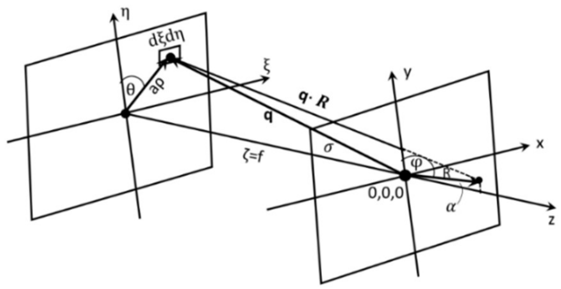

Fig. 7: The relationship between a point on the aperture plane and a point on the diffraction plane.

The ray (r_{12}) from a point on the diffraction plane to a point in the aperture plane is given by the dot product of $\boldsymbol{Q}$ and $\boldsymbol{R}$ such that:

$$r_{12} = \boldsymbol{Q}\cdot\boldsymbol{R} = \xi x + \eta y + \zeta z \quad (63)$$

Substituting the definitions from Eqn. (62), we get:

$$\boldsymbol{Q}\cdot\boldsymbol{R} = a\rho\sin\theta\, R\sin\varphi + a\rho\cos\theta\, R\cos\varphi + \zeta\frac{\kappa R\cos}{\pi} \quad (64)$$

After rearranging and multiplying by k, we get:

$$k\boldsymbol{Q}\cdot\boldsymbol{R} = ka\rho R(\sin\theta\sin\varphi) + ka\rho R(\cos\theta\cos\varphi) + k\zeta\frac{\kappa R\cos\alpha}{\pi} \quad (65)$$

Letting $v = kaR$ and $u = \frac{k\zeta R}{\pi}$, Eqn. (65) becomes:

$$k\boldsymbol{Q}\cdot\boldsymbol{R} = v\rho(\sin\theta\sin\varphi) + v\rho(\cos\theta\cos\varphi) + u\kappa\cos\alpha \quad (66)$$

After simplifying, we get:

$$k\boldsymbol{Q}\cdot\boldsymbol{R} = v\rho\cos(\theta-\varphi) + u\kappa\cos\alpha = kr_{12} \quad (67)$$

After substitution of Eqn. (67) into Eqn. (38), and assuming that the source is far from the aperture, we get:

$$I_2 = \left|\frac{2ikA_o\cos\beta}{4\pi r'_{01}r'_{12}}\iint_{S_{ia}} e^{ik(r_{o1}+r_{12})}\,dS_{ia}\right|^2$$

$$= \left|\frac{2ikA_o\cos\beta e^{ik(r_{o1})}}{4\pi r'_{01}r'_{12}}\iint_{S_{ia}} e^{i(v\rho\cos(\theta-\varphi)+u\kappa\quad)}\,dS_{ia}\right|^2 \quad (68)$$

After expanding the exponentials, we get:

$$I_2 = \left|\frac{2ikA_o\cos\beta e^{ik(r_{o1})}}{4\pi r'_{01}r'_{12}}\left[\iint_{S_{ia}} e^{i(v\rho\cos(\theta-\varphi))}\,e^{i(u\kappa\cos\alpha)}\,dS_{ia}\right]\right|^2 \quad (69)$$

Along the optic axis, $\rho = 0$. Consequently $e^{i(v\rho\cos(\theta-\varphi))} = 1$ and the intensity of the axial diffraction pattern is given by:

$$I_2 = \left|\frac{2ikA_o\cos\beta e^{ik(r_{o1})}}{4\pi r'_{01}r'_{12}}\left[\int_{=0}^{\kappa=1}\int_{\alpha=0}^{\alpha=2\pi} e^{i(u\kappa\cos\alpha)}\,\kappa d\kappa\, d\theta\right]\right|^2 \quad (70)$$

Since $\int_0^{2\pi} e^{i(u\kappa\cos\quad)}\,d\theta = 2\pi J_o(u\kappa)$, we get:

$$I_2 = \left|\frac{2ikA_o\cos\beta e^{ik(r_{o1})}}{4\pi r'_{01}r'_{12}}\left[\int_{\kappa=0}^{\kappa=1} 2\pi J_o(u\kappa)\,\kappa d\kappa\right]\right|^2 \quad (71)$$

Taking the recurrence properties of the Bessel function as given in Eqn. (60) into consideration, the intensity of the axial diffraction pattern is given by:

$$I_2 = \left|\frac{2ikA_o\cos\beta e^{ik(r_{o1})}}{4\pi r'_{01}r'_{12}}\left[\frac{2\pi J_1(u\kappa)}{u\kappa}\right]\right|^2 \quad (72)$$

which has the same functional form as the equation for the lateral diffraction pattern of a circular aperture. In the focal plane, $\kappa = 0$. Consequently, $e^{i(u\kappa\cos\alpha)} = 1$ and the intensity of the diffraction pattern in the focal plane, given by Eqn. (69), is

$$I_2 = \left|\frac{2ikA_o\cos\beta e^{ik(r_{o1})}}{4\pi r'_{01}r'_{12}}\left[\int_{\rho=0}^{\rho=1}\int_{\theta-\varphi=0}^{\theta-\varphi=2\pi} e^{i(v\rho\cos(\theta-\varphi))}\,\rho d\rho\, d\theta\right]\right|^2 \quad (73)$$

Because the Bessel function is symmetrical around the optic axis, we let $\varphi = 0$ without loss of generality. Since $\int_{\theta=0}^{\theta=2\pi} e^{i(v\rho\cos(\theta))}\,d\theta = 2\pi J_o(v\rho)$, we get:

$$I_2 = \left| \frac{2ikA_o \cos\beta e^{ik(r_{o1})}}{4\pi r'_{o1} r'_{12}} \left[\int_{\rho=0}^{\rho=1} 2\pi J_o(v\rho)\, \rho d\rho \right] \right|^2 \quad (74)$$

Taking the recurrence properties of the Bessel function into consideration, the intensity of the lateral diffraction pattern is given by:

$$I_2 = \left| \frac{2ikA_o \cos\beta e^{ik(r_{o1})}}{4\pi r'_{o1} r'_{12}} \left[\frac{2\pi J_1(v\rho)}{v\rho} \right] \right|^2 \quad (75)$$

Eqn. (75) describes and explains the "spurious diameters" of objects viewed with optical instruments by Huygens [78], Herschel [79], Airy [80], and others since then. In general, the intensity of the three-dimensional diffraction pattern is given by:

$$I_2 = \left| \frac{2ikA_o \cos\beta e^{ik(r_{o1})}}{4\pi r'_{o1} r'_{12}} \left[\frac{2\pi J_1(v\rho)}{v\rho} \right] \left[\frac{2\pi J_1(u\kappa)}{u\kappa} \right] \right|^2 \quad (76)$$

where both the lateral and the axial diffraction pattern are formally modelled by the same first-order Bessel function of the first kind with different arguments. Since $v = kaR$ and $u = \frac{k\zeta R}{\pi}$, then $\frac{u}{v} = \frac{k\zeta R}{\pi kaR} = \frac{\zeta}{\pi a} = \frac{f}{\pi a} \cong \frac{n}{\pi\, n\, \text{si}} = \frac{n}{\pi NA}$ and

$$u = v \frac{n}{\pi NA} \quad (77)$$

Eqn. (76) can then be written as:

$$I_2 = \left| \frac{2ikA_o \cos\beta e^{ik(r_{o1})}}{4\pi r'_{o1} r'_{12}} \left[\frac{2\pi J_1(v\rho)}{v\rho} \right] \left[\frac{2\pi J_1(v\left[\frac{\pi n}{NA}\right]\kappa)}{v\left[\frac{\pi n}{NA}\right]\kappa} \right] \right|^2 \quad (78)$$

where the intensities in the lateral and axial diffraction patterns are both described by the same first-order Bessel function of the first kind. However, the argument for the first-order Bessel function of the first kind in the axial part of the equation is increased by π to account for the astigmatic nature of the binary photon and modified by $\frac{n}{NA}$ to account for the properties of the optical system that produces the diffraction pattern. The lateral and axial diffraction patterns predicted by Eqn. (78) are illustrated in Fig. 9. The ellipsoidal shape of the diffraction pattern is consistent with the elongated structure observed in the bright central portion of the diffraction image in a telescope [80] and in light and electron microscopes [81] that is responsible for the depth-of-focus which allows tolerance in finding the focal plane.

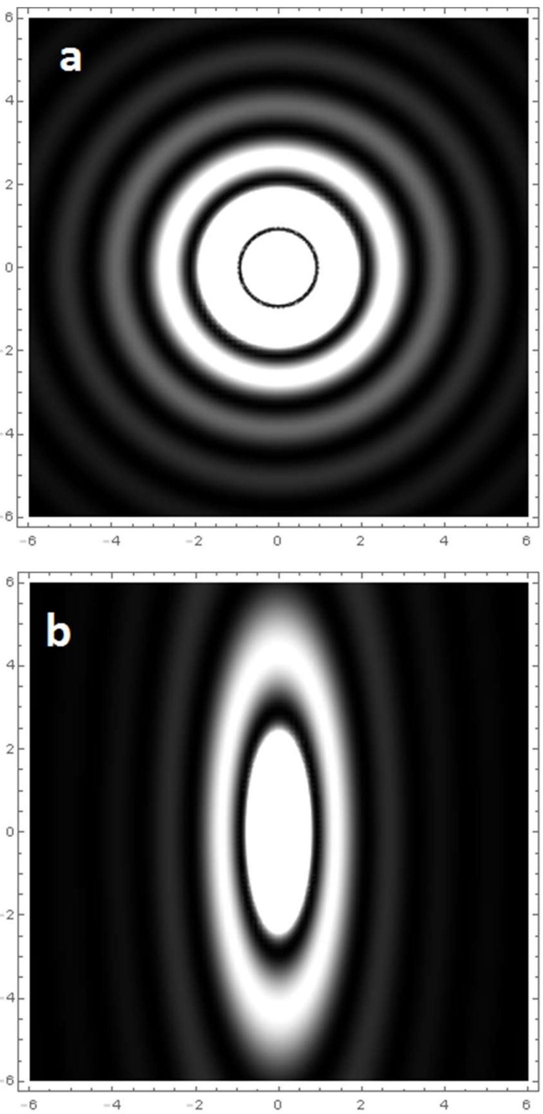

Fig. 9: Illustration of the lateral (a) and axial (b) diffraction patterns predicted by Eqn. 78.

5. The description of plane waves

The functions $U\frac{\partial G}{\partial n}(P)$ and $G\frac{\partial U}{\partial n}(P)$ can also be used to describe the propagation of binary photons along parallel rays, which can be characterized as an electromagnetic plane wave, where the electrical and the magnetic components of light are a quadrature out-of-phase. This description provides a solution to the standard form of Maxwell's [26] wave equation:

$$\nabla^2\Psi = \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} \quad (79)$$

For binary photons traveling in the z direction along parallel rays, let Eqn. (8a) become $U\frac{\partial G}{\partial n}(P) = \sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega\ )}$ and Eqn. (8b) become $G\frac{\partial U}{\partial n}(P) = i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)}$. Then assume that a solution to the wave equation is described by the following complex wave function:

$$\Psi = \sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega\ )} \quad (80)$$

where the complex "wave" consists of a magnetic wave and an electric wave that are a quadrature out-of-phase with each other in Euclidean space and Newtonian time, as predicted by the binary photon model. Plane waves are usually described as having planar surfaces with constant phase. Here we have to describe plane waves as having a planar surface with constant phase for two waves that are quadrature out-of-phase. In order to prove that Eqn. (80) is a solution to the plane wave equation, we have to take the second spatial and temporal derivatives of Ψ. The first spatial derivative of Ψ is:

$$\frac{\partial\Psi}{\partial z} = ik\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i^2k\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} \quad (81)$$

And the second spatial derivative of Ψ is:

$$\nabla^2\Psi = \frac{\partial}{\partial z}\frac{\partial\Psi}{\partial z} = i^2k^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i^3k^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} \quad (82)$$

The first temporal derivative of Ψ is:

$$\frac{\partial\Psi}{\partial t} = -\mathrm{i}\omega\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i^2\omega\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} \quad (83)$$

And the second temporal derivative of Ψ is:

$$\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial}{\partial t}\frac{\partial\Psi}{\partial t} = i^2\omega^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i^3\omega^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} \quad (84)$$

Setting $\nabla^2\Psi = \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}$, we get:

$$i^2k^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i^3k^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} = \frac{1}{c^2}\left[i^2\omega^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i^3\omega^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)}\right] \quad (85)$$

After cancelling $e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)}$, we get

$$i^2k^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i^3k^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}} = \frac{1}{c^2}\left[i^2\omega^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i^3\omega^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right] \quad (86)$$

After simplifying, we get:

$$k^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + ik^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}} = \frac{1}{c^2}\left[\omega^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}} + i\omega^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}\right] \quad (87)$$

After solving the real and imaginary parts separately, we get:

$$k^2\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o}} = \frac{\omega^2}{c^2}\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o}} \quad (88a)$$

$$ik^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2}} = \frac{i\omega^2}{c^2}\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2}} \quad (88b)$$

After cancelling i in Eqn. (88b), we get:

$$k^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2}} = \frac{\omega^2}{c^2}\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2}} \quad (89)$$

We see that Ψ is a solution to the plane wave equation as long as $c^2 = \frac{\omega^2}{k^2}$ for both the real part and the imaginary part. The intensity (I) of the plane wave is obtained by taking the product of the wave function (Ψ) and its complex conjugate (Ψ*):

$$\mathrm{I} = \Psi\Psi^* = \left|\sqrt{\frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2k^2}}e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\omega t)}\right|^2 = \frac{c\boldsymbol{B_o}\cdot\boldsymbol{B_o}}{2\mu_o} + \frac{c\varepsilon_o\boldsymbol{E_o}\cdot\boldsymbol{E_o}}{2} \quad (90)$$

When the emitter and observer are not in the same inertial frame, the electric and magnetic components undergo Dopplerization [82-87] and we get the relativistic second-order wave equation:

$$cc'\nabla^2\Psi = c\frac{\omega_{source}}{k_{observer}}\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\nabla^2\Psi = \frac{\partial^2\Psi}{\partial t^2} \quad (91)$$

where $c' = \frac{\omega_{source}}{k_{observer}}\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}} = \frac{\omega_{source}}{k_{observer}}\frac{1+\frac{v\cos\vartheta}{c}}{\sqrt{1-\frac{v^2\cos^2\vartheta}{c^2}}}$.

ϑ is defined as the angle subtended by light rays extending from the source to the observer and the velocity (v) vector that ends at the observer. As the relative velocity increases, $k_{observer}$ decreases when $\vartheta < \frac{\pi}{2}$, and $k_{observer}$ increases when $\vartheta > \frac{\pi}{2}$. As a result, cc' remains constant in any and all inertial frames [31]. The complex wave solution to the above wave equation is:

$$\Psi = \sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\,\mathrm{co}}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} + i\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\,\mathrm{co}}}\omega t)} \quad (92)$$

where the amplitudes of the electric and magnetic components remain constant with velocity ($v\cos\vartheta$) but the wave number varies with the velocity ($v\cos\vartheta$) of the observer. To show that Eqn. (92) is a solution to Eqn. (91), take the second spatial and temporal derivatives of Eqn. (92):

$$\nabla^2\Psi = i^2k^2\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} + i^3k^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} \quad (93a)$$

$$\frac{\partial^2\Psi}{\partial t^2} = i^2\omega^2\left(\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\right)^2\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} + i^3\omega^2\left(\frac{\sqrt{c+v\,\mathrm{co}}}{\sqrt{c-v\,\mathrm{co}}}\right)^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} \quad (93b)$$

Substitute Eqns. (93a) and (93b) into Eqn. (91) to get:

$$c\frac{\omega_{source}}{k_{observer}}\frac{\sqrt{c+v\,\mathrm{co}}}{\sqrt{c-v\cos\vartheta}}\left[i^2k^2\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} + i^3k^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)}\right] =$$

$$i^2\omega^2\left(\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\right)^2\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} + i^3\omega^2\left(\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\,\mathrm{co}}}\right)^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\, e^{i\left(\boldsymbol{k}\cdot\boldsymbol{z}-\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\omega t\right)} \quad (94)$$

After cancelling like terms, we get:

$$\frac{\omega_{source}}{k_{observer}}\left[k^2\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}} + ik^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\right] = \omega^2\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}} + i\omega^2\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}}\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}} \quad (95)$$

After solving the real and imaginary parts separately, we get:

$$c\frac{\omega_{source}}{k_{observer}}\left[k^2\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}}\right] = \omega^2\frac{\sqrt{c+v\,\mathrm{co}}}{\sqrt{c-v\cos\vartheta}}\sqrt{\frac{c\boldsymbol{B}_o\cdot\boldsymbol{B}_o}{2\mu_o k^2}} \quad (96a)$$

$$\frac{\omega_{source}}{k_{observer}}\left[ik^2\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}}\right] = i\omega^2\frac{\sqrt{c+v\,\mathrm{co}}}{\sqrt{c-v\cos\vartheta}}\sqrt{\frac{c\varepsilon_o\boldsymbol{E}_o\cdot\boldsymbol{E}_o}{2k^2}} \quad (96b)$$

Ψ is a solution to the plane wave equation as long as $c\frac{\omega_{source}}{k_{observer}}\frac{\sqrt{c-v\cos\vartheta}}{\sqrt{c+v\cos\vartheta}} = \frac{\omega^2}{k^2}$ for both the real part and the imaginary part. When $v = 0$ or $\vartheta = \frac{\pi}{2}$, $\frac{\omega_{source}}{k_{observer}} = \frac{\omega^2}{ck^2} = c$ and Eqn. (91) reduces to Eqn. (79). The relationship between the wave number of the electric and magnetic components observed by an observer moving in an inertial frame relative to the wave number of the electric and magnetic components observed by an observer at rest with respect to the source is:

$$k_{source} = k_{observer}\frac{\sqrt{c+v\cos\vartheta}}{\sqrt{c-v\cos\vartheta}} = k_{observer}\frac{1+\frac{v\cos\vartheta}{c}}{\sqrt{1-\frac{v^2\cos^2\vartheta}{c^2}}} \quad (97)$$

which is the equation for the relativistic Doppler effect for the binary photon.

6. Conclusions

The binary photon provides a way to understand with Euclidean space and Newtonian time, why particles with charge and/or a magnetic moment cannot exceed the speed of light [31]. The binary photon also provides a way to understand the deflection of starlight in Euclidian space and Newtonian time [32,84]. Here I have shown that the binary photon naturally provides the twoness required to satisfy the boundary conditions upon which Kirchhoff's diffraction integral in based without calling upon imaginary space as Sommerfeld (1964,2004) chose to do.

In 1861, Maxwell gave the property of twoness to the ether (Fig. 10). Maxwell [88,89] described the ether in terms of electric and magnetic particles: "*According to our hypothesis, the magnetic medium is divided into cells, separated by partitions formed of a stratum of particles which play the part of electricity. When the electric particles are urged in any direction, they will, by their tangential action on the elastic substance of the cells, distort each cell, and call into play an equal and opposite force arising from the elasticity of the cells. When the force was removed, the cells will recover their form, and the electricity will return to its former position*."

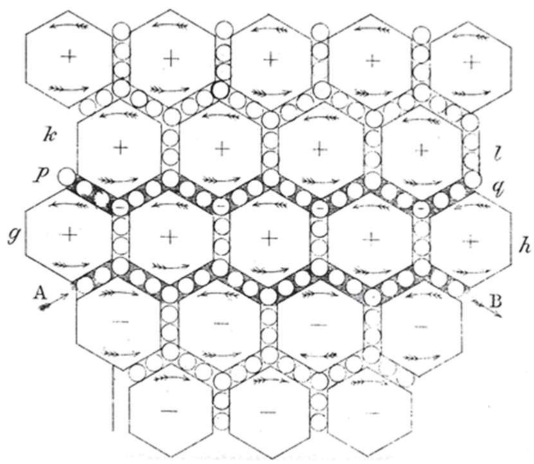

Fig. 10: Maxwell's conception of the luminiferous ether using a mechanical analogy. Notice the similarity between the electric and magnetic components in Maxwell's mechanical ether and the electric and magnetic components in the binary photon [34].

In 1865, Maxwell [90] concluded that "*light and magnetism are affectations of the same substance, and that light is an electromagnetic disturbance propagated though the field according to electromagnetic laws.*" However, by investing some of the properties of electricity and magnetism in the ether, Maxwell had to divest light of those electric and magnetic properties in order to produce his electromagnetic wave theory of light. He did this by assuming that the divergence of the electric and magnetic fields vanished. This led directly to the conclusion that for light, which is electrically neutral, the electric and magnetic fields are in-phase.

Henri Poincaré [23] and Arnold Sommerfeld [24,25] realized the conclusion that the electric and magnetic components are in-phase nullifies the assumptions upon which Kirchhoff's diffraction integral was based and thus Kirchhoff's integral lacked mathematical rigor. Sommerfeld saved the phenomena of the electric and magnetic components being in-phase by introducing imaginary space. Sommerfeld was then able to recover Kirchhoff's diffraction integral based on his own first principles, which were based on the mathematical rigor of imaginary space. Sommerfeld [91] wrote, "*we can confirm the results of the older theory, while we must declare as completely incorrect the methods through which they were derived*." Perhaps the introduction of imaginary space also lacks rigor. Olivier Darrigol [92] wrote that "*From Newton to the present, the diffraction problem has never ceased to be a ground for conflict between physical intuition and mathematical rigor*" and Richard Barakat [93] wrote, "*One of the fundamental unsolved problems of optical diffraction theory is to understand why the Kirchhoff theory successfully predicts the intensity distributions in spite of the fact that from the mathematical standpoint the Kirchhoff theory appears to be a poor approximation to the rigorous formulation of the diffraction problem….*"

Here I show that the solution lies in the binary photon. By making the binary photon electrically neutral as a result of being composed of two equal and opposite electric charges [33,34], the electric and magnetic components of light are quadrature out-of-phase and the boundary conditions for Kirchhoff's diffraction integral are satisfied without the need for a mechanical ether. Thus image formation, which involves the diffraction of light by the specimen and the optical system, becomes completely comprehensible and mathematically rigorous in terms of the electromagnetic properties of light.

References

[1] G. Green, An essay on the application of mathematical analysis to the theories of electricity and magnetism (Printed by T. Wheelhouse, Nottingham, England, 1828). https://books.google.com/books/about/An_Essay_on_the_Application_of_Mathemati.html?id=3444AQAAMAAJ

[2] G. G. Stokes, Transactions of the Cambridge Philosophical Society **9**, 1 (1849).

[3] J. C. Maxwell, Transactions of the Cambridge Philosophical Society **10**, 27 (1855-1856).

[4] W. Thomson, Journal für die reine und angewandte Mathematik von Crelle **39**, 73 (1850).

[5] G. Green, Journal für die reine und angewandte Mathematik von Crelle **39**, 73 (1850).

[6] G. Green, Journal für die reine und angewandte Mathematik von Crelle **44**, 356 (1852).

[7] G. Green, Journal für die reine und angewandte Mathematik von Crelle **47**, 161 (1854).

[8] H. Helmholtz, Journal für Mathematik **57**, 1 (1859).

[9] G. Kirchhoff, Annalen der Physik und Chemie **18**, 663 (1883).

[10] G. Kirchhoff, Mathematische Optik In: *Vorlesungen über mathematische Physik*, Vol 2 (B. G. Teubner, Leipzig, 1891). pp. 64, 121-133.

[11] P. Drude, *Theory of Optics* (Longmans, Green and Co., London, 1939).

[12] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).

[13] E. M. Meyer, *The Diffraction of Light, X-rays, and Material Particles: An Introductory Treatment*, second revised edition (J. W. Edwards, Ann Arbor, MI, 1949).

[14] B. B. Baker and E. T. Copson, *The Mathematical Theory of Huygens' Principle* (Clarendon Press, Oxford, 1950).

[15] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, New York, 1962).

[16] W. C. Elmore and M. A. Heald, *Physics of Waves* (Dover, New York, 1969).

[17] F. A. Jenkins and H. E. White, *Fundamentals of Optics*, fourth edition (McGraw-Hill, New York, 1976).

[18] E. Hecht and A. Zajac, *Optics* (Addison-Wesley, Reading, MA, 1974).

[19] M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, sixth edition (Pergamon Press, Oxford, 1980).

[21] S. G. Lipson and H. Lipson, *Optical Physics*, second edition (Cambridge University Press, Cambridge, 1981).

[22] M. A. Heald and J. B. Marion, *Classical Electromagnetic Radiation*, Third edition (Brooks/Cole, Australia, 1995).

[23] H. Poincaré, *Théorie Mathématique de la Lumière*. Vol II (Gauthier-Villars, Paris, 1892). https://books.google.com/books?id=MOT1z51fSkYC&pg=PR3&lpg=PR3&dq=theorie+mathematique+de+la+lumiere#v=onepage&q=theorie%20mathematique%20de%20la%20lumiere&f=false

[24] A. Sommerfeld, *Mathematical Theory of Diffraction* (Birkhäuser, Boston, 2004).

[25] A. Sommerfeld, *Optics* (Academic Press, New York, 1964).

[26] J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol 2, first edition (Clarendon Press, Oxford, 1873).

[27] H. Weber, Mathematische Annalen **1**, 1 (1869).

[28] H. S. Carslaw, Proceedings of the London Mathematical Society **30**, 121 (1899). http://plms.oxfordjournals.org/content/s1-30/1/121.full.pdf+html

[29] H. A. Bethe, Physical Review **66**, 163 (1944).

[30] D. A. B. Miller, Optics Letters **16**, 1370 (1991).

[31] R. Wayne, Acta Physica Polonica B **41**, 2297 (2010).

[32] R. Wayne, African Review of Physics **7**, 183 (2012).

[33] R. O. Wayne, "Nature of light from the perspective of a biologist: What Is a Photon?," pp. 17-51, in: *Handbook of Photosynthesis*, third edition, Ed. M. Pessarakli (CRC Press, Boca Raton, 2016).

[34] R. Wayne, *A description of the electromagnetic fields of a binary photon* submitted.

[35] H. J. J. Braddick, *Vibrations, Waves, and Diffraction* (McGraw-Hill Book Co., New York, 1965).

[36] G. Bekefi and A. H. Barrett, *Electromagnetic Vibrations, Waves, and Radiation* (MIT Press, Cambridge, MA, 1977).

[37] P. M. Duffieux, *The Fourier Transform and Its Applications to Optics*, Second edition (John Wiley & Sons, New York, 1983).

[38] R. G. Wilson, *Fourier Series and Optical Transform Techniques in Contemporary Optics: An Introduction* (John Wiley & Sons, New York, 1995).

[39] J. W. Goodman, *Introduction to Fourier Optics*, Second edition (McGraw-Hill Book Co., New York, 1996).

[40] A. Fresnel, Memoir on the diffraction of light, pp. 79-144, in: *The Wave Theory: Light and Spectra*, Ed. H. Crew (Arno Press, New York, 1981).

[41] A. Fresnel, Fresnel's prize memoir on the diffraction of light, pp. 147-176, in: *Classical and Modern Diffraction Theory*, Eds. K. Klem-Musatov, H. C. Hoeber, T. J. Moser and M. A. Pelissier (Society of Exploration Geophysicists, Tulsa, OK, 2016).

[42] M. W. Jackson, *Spectrum of Belief: Joseph von Fraunhofer and the Craft of Precision Optics* (MIT Press, Cambridge, MA, 2000).

[43] K. Hentschel, *Mapping the Spectrum: Techniques of Visual Representation in Research and Teaching* (Oxford University Press, Oxford, 2002).

[44] B. R. A. Nijboer, *The Diffraction Theory of Aberrations*, Ph.D. dissertation (University of Groningen, Groningen, The Netherlands, 1942). http://www.nijboerzernike.nl/

[45] H. Helmholtz, Annalen der Physik und Chemie **6**, 557 (1874).

[46] E. Abbe, Archiv für mikroskopische Anatomie **9**, 413 (1873).

[47] P. J. Shaw and D. J. Rawlins, Journal of Microscopy **163**, Pt. 2, 151 (1991).

[48] H. E. Keller, Objective lenses for confocal microscopy, pp.111-126, in: *Handbook of Biological Confocal Microscopy*, Ed. J. B. Pawley (Plenum Press, New York, 1995).

[49] T. Li, S. Ota, J. Kim, Z. J. Wong, X. Yin and X. Zhang, Scientific Reports **4**, 7253 /(2014).

[50] F. M. Schwerd, *Die Beugungserscheinungen aus den Fundamentalgesetzen der Undulationstheorie Analytisch Entwinkelt und in Bildern Darsestellt* (Schwan und Goetz'schen, Mannheim, 1835).

[51] G. B. Airy, Transactions of the Cambridge Philosophical Society **5**, 283 (1835).

[52] E. Lommel, *Studien über die Bessel'schen Functionen* (B. G. Teubner, Leipzig, 1868)

[53] A. Gray and G. B. Mathews, *Bessel Functions and their Applications to Physics* (Macmillan, London, 1895)

[54] G. N. Watson, *A Treatise on the Theory of Bessel Functions, Second edition* (Cambridge University Press, Cambridge, 1966).

[55] NIST, *Digital Library of Mathematical Functions: Bessel functions; Spherical Bessel Functions* http://dlmf.nist.gov/10.2; http://dlmf.nist.gov/10.47.

[56] E. Lommel, Die Beugungserscheinungen einer kreisrunden Oeffnung und einses kreisrunden Schirmchens: theoretisch und experimentel. Abhandlungen der mathematisch-physikalischen Classe der Königlich Bayerischen Akademie der Wissenschaften **XV** (II), 229 (1886).

[57[E. Lommel, Die Beugungserscheinungen geradling begrenzter Schirme Abhandlungen der mathematisch-physikalischen Classe der Königlich Bayerischen Akademie der Wissenschaften **XV** (III), 529 (1886).

[58] H. Struve, Mémoires de l'Acaéme Impériale des Sciemces de St. Pétersbourg Series 7, **30** (8), 1 (1882).

[59] K. Schwarzchild, Sitzungsberichte der mathematisch-physikalischen Classe der k. b. Akademie der Wissenschaften zu München **28**, 271, (1899).

[60] P. Debye, Annalen der Physik **30**, 755 (1909).

[61] M. Berek, Z. Phys. **40**, 421 (1926).

[62] G. C. Steward, Philosophical Transactions of the Royal Society of London A **225**, 131 (1926).

[63] G. C. Steward, *The Symmetrical Optical System* (Cambridge University Press, Cambridge, 1928).

[64] J. Picht, *Optische Abbildung: Einführung in die Wellen- und Beugungstheorie optischer Systeme* Springer, Weisbaden, 1931).

[65] F. Zernike and B. R. A. Nijboer, Théorie de la diffraction des aberration, pp. 227-235, in: La Théorie des Images Optiques (Éditions de la Revue D'Optique, Paris, 1949).

[66] E. Wolf, Proc. Roy. Soc. A **204**, 533 (1951).

[67] E. H. Linfoot and E. Wolf, Monthly Notices of the Royal Astronomical Society **112**, 452 (1952).

[68] E. H. Linfoot and E. Wolf, Proceedings of the Physical Society B **66**, 145 (1953).

[69] E. H. Linfoot, *Recent Advances in Optics* (Clarendon Press, Oxford, 1955).

[70] E. H. Linfoot and E. Wolf, Proceedings of the Physical Society B **69,** 823 (1956).

[71] C. A. Taylor and B. J. Thompson, Journal of the Optical Society of America **48** (11), 844 (1958).

[72] C. W. McCutchen, Journal of the Optical Society of America **54**, 240 (1964).

[73] H. Riesenberg, Optisches System des Mikroskops pp. 24-107 in: Handbuch der Mikroskopie, Ed. H. Riesenberg (VEB Verlag Technik, Berlin, DDR 1988).

[74] M. Pluta, *Advanced Light Microscopy. Volume 1: Principles and Basic Properties* (Elsevier, Amsterdam, 1988).

[75] T. Wilson, Ed. *Confocal Microscopy* (Academic Press, London, 1990).11, 187 (2016).

[76] C. W. McCutchen, Journal of the Optical Society of America A **8**, 868 (1991).

[77] S. Inoué, Foundations of confocal scanned imaging in light microscopy, pp.1-14, in: *Handbook of Biological Confocal Microscopy*, Ed. J. B. Pawley (Plenum Press, New York, 1995).

[78] A. E. Bell, *Christian Huygens and the Development of Science in the Seventeenth Century* (Edward Arnold & Co., London, 1947).
[79] W. Herschel, Philosophical Transactions of the Royal Society of London **95**, 31 (1805).
[80] H. D. Taylor, Monthly Notices of the Royal Astronomical Society **54**, 67 (1893).
[81] E. M. Slayter, *Optical Methods in Biology* (John Wiley & Sons, New York, 1970).
[82] R. Wayne, African Physical Review **4**, 43 (2010).
[83] R. Wayne, African Review of Physics **8**, 283 (2013).
[84] R. Wayne, African Review of Physics **11**, 187 (2016).
[85] A. F. Maers and R. Wayne, African Physical Review **5**, 7 (2011).
[86] A. Maers, R. Furnas, M. Rutzke and R. Wayne, African Review of Physics **8**, 297 (2013).
[87] A. Maers, R. Furnas, M. Rutzke and R. Wayne, African Review of Physics **9**, 536 (2014).
[88] J. C. Maxwell, On physical lines of force, pp. 450-513, in: *The Scientific Papers of James Clerk Maxwell*. Volume 1, Ed. W. D. Niven, (Dover, Mineola, NY, 2003).
[89] O. Lodge, *Modern Views of Electricity* Third revised edition (Macmillan, London, 1907).
[90] J. C. Maxwell, A dynamical theory of the electromagnetic field, pp. 526-597, in: *The Scientific Papers of James Clerk Maxwell*. Volume 1, Ed. W. D. Niven, (Dover, Mineola, NY, 2003).
[91] S. Seth, *Crafting the Quantum: Arnold Sommerfeld and the Practice of Theory, 1890-1926* (MIT Press, Cambridge, MA, 2010).
[92] O. Darrigol, *A History of Optics: From Greek Antiquity to the Nineteenth Century* (Oxford University Press, Oxford, 2012).
[93] R. Barakat, Progress in Optics **1**, 67 (1961).

Received: 11 September, 2018
Accepted: 05 July, 2019