Using the Schrödinger Equation for a Boson to Relate the Wave-like Qualities and Quantized Particle-like Quantities of the Binary Photon in Euclidean Space and Newtonian Time

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The photon is an enigmatic entity that is often described in terms of its wave-particle duality. I demystify the wave-particle duality in the model of a binary photon by claiming that the fundamental unit of light is not an elementary particle, but is a composite entity composed a particle and its conjugate antiparticle. These semi-photons rotate around the axis of propagation in the transverse plane as they oscillate and translate along the axis of propagation. I developed wave functions that represent the paths of the semi-photons in Euclidean space and Newtonian time. The three dimensional wave functions can be resolved into the wave functions that describe the paths of the semi-photons in a plane transverse to the axis of propagation and the wave functions that describe the paths of the semi-photons parallel to the axis of propagation. These wave functions are solutions to the Schrödinger equation that has been modified to directly operate on bosons as opposed to fermions. Using the modified Schrödinger equation as well as the standard equations of mechanics, I have obtained eigenvalues for the rotational, vibrational, and translational energies as well as for the angular and linear momenta of the binary photon. The eigenvalues for the mechanical properties of the binary photon with total energy of $\hbar\omega$ are $\frac{\hbar\omega}{2}$, 0, $\frac{\hbar\omega}{2}$, $\hbar$, and $\hbar k$, respectively. The wave mechanical approach presented here shows that the binary photon can be visualized as an oscillating rotor propagating through Euclidean space and Newtonian time. While quantum mechanical calculations typically agree with experience while being at odds with ordinary concepts of trajectories in space and time, the wave mechanical calculations carried out here agree with experience without conflicting with the ordinary concepts of space and time. Indeed, in contrast to the claims of Heisenberg and Born, the mathematical description of the quantized binary photon presented here is consistent with the *Anschaulichkeit*, picturability, or imaged facts of classical physics sought by Einstein.

"*It is even undeniable that there is an extensive group of facts concerning radiation that shows that light possesses certain fundamental properties that can be understood far more readily from the standpoint of Newton's emission theory of light than from the standpoint of the wave theory. It is therefore my opinion that the next stage in the development of theoretical physics will bring us a theory of light that can be understood as a kind of fusion of the wave and emission theories of light.*"

--A. Einstein [1]

1. Introduction

The concept of discontinuous packets of energy or quanta emerged from Planck's analysis of simple harmonic oscillators with one degree of freedom[1] that were in thermal equilibrium with blackbody radiation [3]. Einstein took the discontinuous nature of energy further and postulated that the light quantum itself was a dimensionless, mathematical point that had both particle-like and wave-like properties [4,5]. The wave-particle duality was applied to matter by de Broglie [6] who suggested that elementary material particles, such as electrons, would also have wave-like properties. In his "double solution," de Broglie postulated that a particle, defined by a wave function (u) that had an infinity, moved along a wavelike track, defined by another wave function (Ψ).

Schrödinger [7] extended de Broglie's view and modeled an elementary particle as a wave packet formed by the interference that results from the superposition of a large number of coherent infinite plane waves. Each wave had its own wave function whose amplitude (A_ν) represented the contribution of a wave of a given frequency (ν) to the resultant wave packet. At the suggestion of Peter Debye [8], Schrödinger constructed an equation of motion in terms of a one-dimensional diffusion equation to characterize the spatial and temporal behavior of the wave packet. A limitation of the Schrödinger equation as a model of the motion of elementary particles is that the wave packet, which is operated upon in Schrödinger's equation, does not hold together like a particle does but disperses over time [9].

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[1] In response to a question from Poincaré at the 1911 Solvay Conference, Planck [2] responded that extending his treatment to more than one degree of freedom "would soon be possible."

While the Schrödinger equation was successful in modeling the spectral lines of hydrogen and was fertile in its applicability to scattering phenomena, its power came at a price. The world of microphysics was no longer visualizable in the same manner that the world of macrophysics was visualizable. This is because the Schrödinger equation is not a wave equation but a diffusion equation[2] where the partial derivatives are second order with respect to space and first order with respect to time. Consequently, the only solutions to Schrödinger's equation are complex wave functions ($\Psi(z,t)$) that contain both real and imaginary parts (Appendix 1). To Schrödinger [7], the use of imaginary numbers was just a "*mathematical device of calculation*" so that the imaginary term could be counted separately from the real term just as apples are counted separately from oranges. By contrast, Pauli [10] asserted that "*the imaginary coefficient assures that there is no special direction in time*." This interpretation of the imaginary term combined with Born's [11] claim that "*Schrödinger's waves move not in ordinary space but in configuration space*" led to the current idea that "*the quantum theory describes the world in terms of an abstract, many-dimensional configuration space, and the number of dimensions is proportional to the total number of particles in the world* [12][3]." The quantum concept of space and time also led to the assertion that causality did not apply to elementary particles, which are the fundamental building blocks of matter found in the world of macrophysics. According to Jordan [12] determinism "*is not itself a natural law; the natural laws are the differential equations*" and "*we can at present compute all probabilities; but we cannot understand any of them*." Schrödinger's equation was too valuable to ignore even if it was unable to give a picture of the atomic world. In a lecture given at Cornell University in the Fall of 1926, Lorentz [15] said, "*It is difficult to attach a clear physical meaning to Schrödinger's wave equation however interesting and fruitful may be the consequences that are drawn from it.*" This dilemma was captured by Felix Bloch [8] in a poem:

Erwin with his psi can do
Calculations quite a few,
But one thing has not been seen:
Just what does psi really mean?

The real and imaginary parts of the wave function $\Psi(z,t)$ were interpreted by Born [16] as a mathematical device to get to $\Psi(z,t)\Psi^*(z,t)$, which is the product of the wave function and its complex conjugate. Born surmised that observable quantities in the real world could only be represented by real numbers. Consequently for him, the wave function, with its mix of real and imaginary numbers, represented the movement of a particle through configuration space that defied all attempts at visualizability, and it did not represent a guide or the path of a particle moving through ordinary space and time. He called the complex wave function a probability amplitude that could not be observed, and the product of the wave function and its complex conjugate, which is a real number, the probability that a particle would be observable at point z at time t. According to the Copenhagen interpretation of quantum mechanics, upon observation, an infinite number of superpositioned wave functions, each of which describes a possible state of a particle in space and time, collapses into one wave function that describes the most probable state of a particle in space and time.

This is the orthodox interpretation of quantum mechanics [17]. It is the interpretation of the consensus of physicists, although other interpretations also exist. According to the many-worlds interpretation of quantum mechanics [18], each of the wave functions that make up a wave packet describes the localization of a particle in space and time in a different world or even a different universe. The realist interpretation, taken by Max Planck, Albert Einstein, Louis de Broglie, and Erwin Schrödinger asserts that quantum mechanics as it was interpreted by Niels Bohr, Max Born, and Werner Heisenberg, is an incomplete theory and that additional information other than that which typically goes into $\Psi(z,t)$ is necessary for a complete description of a particle in space and time [19-23]. I too am a realist.

While Schrödinger's equation was developed in analogy to Hamilton's equation for light rays, it has

[2] The Schrödinger equation can be transformed into a typical diffusion equation by letting $\frac{\hbar}{2m} \to D$, the diffusion coefficient and letting $t \to -it$. The diffusion equation is called the heat equation when it describes the diffusion of thermal photons.

[3] Lorentz and Kennard realized the relation between a representation in ordinary space and one in configuration space. Lorentz [13] wrote "*To represent the motion of a system of n material points, one can of course make use of a space of 3 dimensions with n points or of a space of 3n dimensions where the systems will be represented by a single point. This must amount to exactly the same thing; there can be no fundamental difference. It is merely a question of knowing which of the two representations is the most suitable, which is the most convenient. But I understand that there are cases where the matter is difficult. If one has a representation in a space of 3n dimensions, one will be able to return to a space of 3 dimensions only if one can reasonably separate the 3n coordinates into n groups of 3, each corresponding to a point, and I could imagine that there may be cases where that is neither natural nor simple. But, after all, it certainly seems to me that all this concerns the form rather than the substance of the theory.*" Kennard [14] wrote, "*We can also, however, replace the n-dimensional packet by n separate packets, one for each particle, all moving in the same ordinary space*."

not been possible heretofore to apply Schrödinger's equation to a photon. Here I show that it is possible to realistically interpret the solutions to the Schrödinger equation as wave functions upon which sub-photonic particles move in Euclidean space and Newtonian time along the axis of propagation and in a plane orthogonal to the axis of propagation. Here I also show that Schrödinger's equation developed for a boson gives the particle-like eigenvalues of the binary photon and its internal constituents that move in ordinary space and time.

I start with the assumption that the photon is not a mathematical point-like elementary particle, but it is an extended object that provides space for internal movements (Fig. 1). Silberstein [24,25], Ornstein and Burger [26], Marx [27], Lehnert [28], and Wayne [29] also posit that the photon has transverse dimensions. Moreover, like Bragg [30], de Broglie [31], Born [32], and Jordan [33], I assume that the binary photon is a composite particle composed of two sub-photonic conjugate particles known as semi-photons. I further posit that the two semi-photons simultaneously oscillate parallel to and rotate around the axis of propagation [34]. Each semi-photon has definite position and momentum in Euclidean space at all times. The semi-photons move in three dimensions relative to the time-averaged center of gravity of the binary photon. As described elsewhere [34], the time-averaged center of gravity of the binary photon propagates at the speed of light (c) as a result of a gravity-dependent Coulombic force between the semi-photons. Here I will present wave functions or eigenvectors that describe the rotations and oscillations of the semi-photons in the rest frame of the binary photon relative to its time-averaged center of gravity. The wave functions define the time-varying, wave-like properties of the binary photon. I will show how the Schrödinger equation for a boson and the classical laws of physics can be used to determine the eigenvalues that are related to transverse rotational motion, and the eigenvalues that are related to the longitudinal translational and vibrational motion. These eigenvalues define the mechanical particle-like properties of the binary photon.

The two semi-photons that make up the monochromatic binary photon are conjugate particles in terms of charge, parity, and mass. This symmetry between matter and antimatter, known as charge-parity-mass (CPM) symmetry [35-37] differs from charge-parity-time (CPT) symmetry in that CPM symmetry is based on the unidirectional and irreversible arrow of Newtonian time in which both matter and antimatter exist. In CPM symmetry, the binary photon is not an elementary particle and its own antiparticle as it is in CPT symmetry but a composite entity composed of a particle of matter and a particle of antimatter [34].

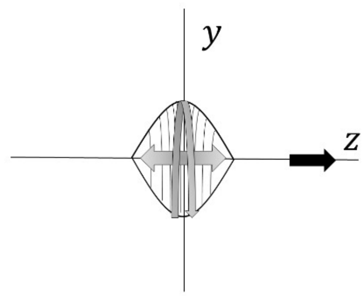

Fig. 1: Simplified model of a binary photon as an extended object. Over one cycle in which the binary photon propagates a distance of one wavelength (λ), the diameter varies between $\frac{\lambda}{2\pi}$ and $\frac{\lambda}{\pi}$ and the length varies between 0 and $\frac{\lambda}{\pi^2}$.

2. Results and Discussion

In the transverse plane, perpendicular to the axis of propagation, each semi-photon moves in its own ring with a constant radius centered on the axis of propagation (Fig. 2). The radius (r) of the ring, within which each semi-photon moves, is a function of the wavelength of the binary photon ($r = \frac{\lambda}{2\pi}$) and is equal to the reciprocal of its wave number ($r = \frac{1}{k}$). The circumference (l) of each ring is equal to the wavelength ($\lambda = l = 2\pi r$). Each semi-photon completes a full orbit around the ring with an angular frequency that is equal to the angular frequency (ω) of the binary photon and related to its frequency (ν) and wavelength ($\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$). Unless the binary photon is acted upon by an external force, the radius is constant and independent of time.

Consider a ring to be a one dimensional path of length l in which a semiphoton moves. The wave function ($\Psi(l,t)$) that describes the movement of a semi-photon around a ring with circumference l must be continuous, finite, and single valued in order to give rise to a time-independent standing wave or stationary state that results in a stable binary photon. As long as m_r is an integer, the complex exponential $\left(e^{im_r\frac{2\pi}{\lambda}d}\right)$ is a function that gives the same value at the same position (d) in the ring no matter how many complete times (m_r) the semiphoton travels around the ring.

As long as we are only discussing the mechanics[4] of the binary photon, we can convert a two-body problem into a one-body problem and simplify the construction of the wave function or eigenvector by combining the motion of two conjugate semi-photons that have masses and senses of rotation with opposite signs (Fig. 3a) into the motion of one binary photon (Fig. 3b), with a mass that is twice the modulus of either semi-photon and moving with the same sense as the positive mass semi-photon. In a period, the combined mass binary photon is considered to travel a distance of one wavelength—the same distance that each semiphoton individually travels in a period.

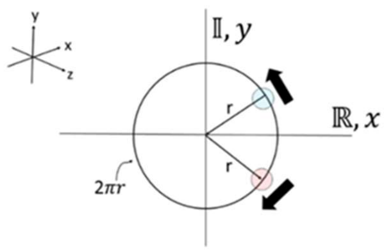

Fig. 2: Anticlockwise rotation of the positive mass semi-photon (blue) and clockwise rotation of the negative mass semi-photon (red) in the transverse plane (xy) perpendicular to the axis of propagation (z). The configuration space is equivalent to ordinary space where the real axis is the x-axis and the imaginary axis is the y-axis. The radius of a ring is equal to the reciprocal of its wave number (k). The circumference of a ring is equal to the wavelength (λ) of the binary photon. Following Descartes, at any given time, the position of a geometric point on a plane can be defined by two numbers using algebra. In the general case, one number can be considered real and the other one imaginary. Here I consider one number (the real number) to correspond to the x-axis and the other number (the imaginary number) to correspond to the y-axis. Each semi-photon orbits with an angular frequency that is equal to the angular frequency (ω) of the binary photon. Note that formally a binary photon could also be composed of a positive mass rotating clockwise and a negative mass rotating anticlockwise. The angular momentum of a binary photon in which the positive mass semi-photon rotates clockwise and the negative mass semi-photon rotates anticlockwise is antiparallel to the direction of propagation while the angular momentum of a binary photon in which the positive mass semi-photon rotates anticlockwise and the negative mass semi-photon rotates clockwise is parallel to the direction of propagation

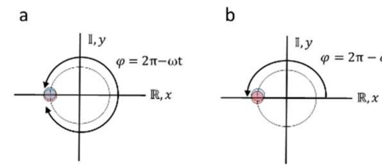

Fig. 3: The binary photon. Two-body problem: (a) one semi-photon with positive mass travels anticlockwise while the conjugate semi-photon with negative mass travels clockwise. In order to create an equivalent one-body problem (b) a binary photon with twice the mass of the positive mass semi-photon travels anticlockwise.

Rotation of a single mass in a single two-dimensional ring is a one-dimensional problem as far as the Schrödinger equation is concerned. The one-dimensional Schrödinger equation for the movement of a free particle along the z-axis is given by:

$$\left(-\frac{\hbar^2}{2m}\right)\frac{\partial^2\Psi(z,t)}{\partial z^2} + \mathrm{V}\Psi(z,t) = \frac{i\hbar}{n}\frac{\partial\Psi(z,t)}{\partial t} \quad (1)$$

where $\hbar$ is the reduced Planck's constant, m is the mass of the particle, i is an imaginary number equal to $\sqrt{-1}$, V is the potential energy, and $\Psi(z,t)$ is the wave function with respect to space (z) and time (t). In the standard version of the Schrödinger equation for fermions, $n = 1$. I was unable to find a realistic wave function to describe the mechanical properties of the binary photon with the Schrödinger equation as long as $n = 1$. Realizing that Schrödinger created his equation for fermions, I modified the Schrödinger equation to take into account bosons by letting $n = 2$. Since the total energy ($E_{total} = \hbar\omega$) of the binary photon can be equipartitioned into transverse rotational energy and longitudinal translational and vibrational energy [34], I postulated separate and independent wave functions to describe the transverse motions and the longitudinal motions of the semi-photons.

The particle with the combined mass traveling around the ring can be treated as it is in quantum chemistry, as a free particle where the potential energy on the ring (V_{ring}) vanishes and the potential energy outside the ring ($V_{outside\ ring}$) is infinite [39]. When VΨ vanishes, the particle moving on the ring has total rotational energy $E_{rotational}$ that is equal to the kinetic energy on the ring (KE_{ring}). So defined, the operator $\left(-\frac{\hbar^2}{2m}\right)\frac{\partial^2}{\partial z^2}$ gives the total rotational energy when

[4] On the other hand, when considering the electrodynamics of the binary photon by taking the charge of the semi-photons into consideration [38], it is perspicacious to show the individual wave functions of the semi-photons explicitly before combining them for concision of expression into a single wave function for the binary photon.

applied to the wave function. Using polar coordinates with a fixed radius that is determined by the wavelength of the binary photon with radius $\left(r = \frac{\lambda}{2\pi}\right)$ and circumference ($l = \lambda$), the Schrödinger equation for a boson for the movement of a particle in a ring becomes [39]:

$$\left(-\frac{\hbar^2}{2m_{binary\ photon}}\right)\frac{\partial^2\Psi(\varphi,t)}{r^2\partial\varphi^2} = \frac{i\hbar}{2}\frac{\partial\Psi(\varphi,t)}{\partial t} \quad (2)$$

where $m_{binary\ pho}$ is the mass of the binary photon. Since the moment of inertia (I) for a mass at the end of a massless string is given by:

$$I = m_{binary\ photon}r^2 \quad (3)$$

and the moment of inertia of a semi-photon of mass ($m_{semiphoto} = \frac{\hbar\omega}{2c^2}$) [34] is given by:

$$I_{semiphoton} = m_{semiphoto}\ r^2 = \frac{\hbar\omega}{2c^2k^2} \quad (4),$$

where $r = \frac{\lambda}{2\pi} = \frac{1}{k}$ is a necessary condition to ensure that the angular momentum of the binary photon is $\pm\hbar$ [34]. The sum of the moments of inertia of a binary photon composed of a semi-photon with a positive mass moving anticlockwise and a semi-photon with negative mass moving clockwise is given by:

$$I_{binary\ pho} = m_{binary\ photon}\ r^2 = \frac{\hbar\omega}{c^2k^2} \quad (5)$$

where $m_{binary\ photo} = 2|m_{semiphoton}|$ [34]. The Schrödinger equation using polar coordinates for the rotational motion of a generalized binary photon of any wavelength in the transverse plane becomes:

$$\left(-\frac{\hbar^2}{2I_{binary\ photon}}\right)\frac{\partial^2\Psi(\varphi,t)}{\partial\varphi^2} = -\frac{i\hbar}{n}\frac{\partial\Psi(\varphi,t)}{\partial t} = \frac{i\hbar}{2}\frac{\partial\Psi(\varphi,t)}{\partial t} \quad (6)$$

where $n = 2$. I postulate that the rotational wave function ($\Psi(\varphi,t)$) and its complex conjugate ($\Psi^*(\varphi,t)$) for a combined particle of any mass representing binary photon of any wavelength moving in a ring are:

$$\Psi(\varphi,t) = Ae^{im_r\varphi}e^{-i\omega t}$$
$$= A[\cos m_r\varphi + i\sin m_r\varphi]e^{-i\omega t} \quad (7a)$$

$$\Psi^*(\varphi,t) = Ae^{-im_r\varphi}e^{i\omega t}$$
$$= A[\cos m_r\varphi - i\sin m_r\varphi]e^{i\omega t} \quad (7b)$$

where m_r is the rotational quantum number for the binary photon, A is the amplitude of the wave function and equals unity since the binary photon is described as a monochromatic wave rather than a wave packet, and ω is the rotational frequency of the combined mass binary photon, which is equal to the angular frequency of the monochromatic binary photon. $\Psi(\varphi,t)$ is the wave function for a binary photon whose combined semi-photon masses rotate anticlockwise[5] ($m_r > 0$). The complex conjugate ($\Psi^*(\varphi,t)$) of the wave function is also identical to a wave function for a binary photon whose combined semi-photon masses rotate clockwise ($m_r < 0$). The ring is two dimensional in that it exists in the xy plane perpendicular to the axis of propagation ($+z$) in Euclidean space. The real axis in the transverse plane is equivalent to the x-axis and the imaginary axis is equivalent to the y-axis. With this picture, ordinary space is equivalent to configuration space and the imaginary number does not represent an unreal quantity but an orthogonal quantity that cannot be summed algebraically with a real quantity. The real and imaginary values must be summed as components of a vector.

Now we must check if the proposed wave function is a solution to the Schrödinger equation for a boson by substituting Eqn. (7a) into Eqn. (6):

$$\left(-\frac{\hbar^2c^2k^2}{2\hbar\omega}\right)\frac{\partial^2 e^{im_r\varphi}e^{-i\omega t}}{\partial\varphi^2} = \frac{i\hbar}{2}\frac{\partial e^{im_r\varphi}e^{-i\omega t}}{\partial t} \quad (8).$$

After differentiating Eqn. (8), we get:

$$\left(-\frac{\hbar^2c^2k^2}{2\hbar\omega}\right)i^2m_r^2e^{im_r\varphi}e^{-i\omega t} = -\frac{i^2\hbar\omega}{2}e^{im_r\varphi}e^{-i\omega t} \quad (9)$$

After simplifying Eqn. (9), we get:

$$m_r^2 = 1 \quad (10).$$

The wave function (Eqn. 7a) for the rotational energy in the transverse plane of the binary photon is a solution to the Schrödinger equation for a boson only if $m_r = \pm 1$. m_r is a dimensionless quantum number equivalent to the bosonic spin quantum number in standard quantum mechanics. While in standard quantum mechanics, the intrinsic spin is a quantum

[5] Anticlockwise and clockwise are defined as the binary photon approaches the observer. With anticlockwise rotation, the thumb of the right hand points in the direction of propagation and the fingers of the right hand curl with the sense of rotation of the positive mass semi-photon or the combined mass binary photon. With clockwise rotation, the thumb of the left hand points in the direction of propagation and the fingers of the left hand curl with the sense of rotation of the positive mass semi-photon or the combined mass binary photon.

number that does not represent a mechanical motion because motion cannot take place within a mathematical point, here the spin represents the sense of the rotational motion intrinsic to the binary photon. In order to find the energy eigenvalue for the intrinsic rotational motion of the binary photon in the transverse plane, we must separate the rotational wave function into its spatial and temporal parts that describe movement in absolute Euclidean space (φ) and Newtonian time (t):

$$\Psi(\varphi,t)=\psi(\varphi)T(t) \quad (11)$$

To find the eigenvalues for a binary photon, substitute $\Psi(\varphi,t)=\psi(\varphi)T(t)$ into the Schrödinger equation for a boson:

$$\left(\frac{-\hbar^2}{2I_{binary\ photo}}\right)\frac{\partial^2[\psi(\varphi)T(t)]}{\partial\varphi^2}=\frac{i\hbar}{2}\frac{\partial[\psi(\varphi)T(t)]}{\partial t} \quad (12)$$

Treat the variables inside the partial derivatives that are not part of the partial derivative as constants:

$$\left(\frac{-\hbar^2}{2I_{binary\ photon}}\right)T(t)\frac{\partial^2[\psi(\varphi)]}{\partial\varphi^2}=\frac{i\hbar}{2}\psi(\varphi)\frac{\partial[T(t)]}{\partial t} \quad (13)$$

Divide by $\psi(\varphi)T(t)$:

$$\left(\frac{-\hbar^2}{2I_{binary\ photo}}\right)\frac{T(t)}{\psi(\varphi)T(t)}\frac{\partial^2[\psi(\varphi)]}{\partial\varphi^2}$$
$$=\frac{i\hbar}{2}\frac{\psi(\varphi)}{\psi(\varphi)T(t)}\frac{\partial[T(t)]}{\partial t} \quad (14)$$

Cancel like terms to get a fully separated equation where one term is only a function of Euclidean space (φ) and the other term is only a function of Newtonian time (t).

$$\left(\frac{-\hbar^2}{2I_{binary\ pho}}\right)\frac{1}{\psi(\varphi)}\frac{\partial^2[\psi(\varphi)]}{\partial\varphi^2}=\frac{i\hbar}{2}\frac{1}{T(t)}\frac{\partial[T(t)]}{\partial t} \quad (15)$$

$E_{rotational}$, the separation constant, is the eigenvalue for the total rotational energy of the binary photon. After separating the variables, the partial differential equation becomes two ordinary differential equations that give $E_{rotational}$:

$$E_{rotational}=\left(\frac{-\hbar^2}{2I_{binary\ photon}}\right)\frac{1}{\psi(\varphi)}\frac{d^2[\psi(\varphi)]}{d\varphi^2} \quad (16a)$$

$$E_{rotational}=\frac{i\hbar}{2}\frac{1}{T(t)}\frac{d[T(t)]}{dt} \quad (16b)$$

Eqn. (16a) is the time-independent Schrödinger equation, which gives the curvature of $\psi(\varphi)$ in the transverse plane. The curvature of the trajectory of the semi-photons described by the wave function is picturable in that the greater the rotational energy of a binary photon is, the greater is the curvature of $\psi(\varphi)$ in the transverse plane. The wave function of highly energetic binary photons will have a great curvature and will make a tight circle in the transverse plane that appears particle-like while the wave function of low energy binary photons will have a small curvature in the transverse plane and will make a wide circle that appears plane wave-like.

Next, we solve Eqn. (16a) to get the eigenvalue of the rotational energy of binary photon:

$$E_{rotational}=\left(-\frac{\hbar^2c^2k^2}{2\hbar\omega}\right)\frac{1}{\psi(\varphi)}\frac{d^2[e^{im_r\varphi}]}{d\varphi^2}$$
$$=\left(-\frac{\hbar^2c^2k^2}{2\hbar\omega}\right)\frac{i^2m_r^2}{\psi(\varphi)}[e^{im_r\varphi}]=\frac{\hbar\omega}{2} \quad (17)$$

Table 1. Operators and Equations to Determine the Eigenvalues for the Mechanical Properties of the Binary Photon

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Mechanical Property	Operator or Equation
Transverse rotational energy	$\hat{H}_{rotational}\equiv\left(-\frac{\hbar^2c^2k^2}{2\hbar\omega}\right)\frac{\partial^2}{\partial\varphi^2}$
Angular momentum	$\hat{L}_z\equiv\frac{\hbar}{i}\frac{\partial}{\partial\varphi}$
Translational energy	$E_{translational}=\frac{1}{2}mv^2$
Linear momentum	$p=\sqrt{2mE_{translational}}$

Note that I am modeling the movement of the combined semi-photons relative to the time-averaged center of gravity with respect to the *z*-axis.

When the wave function operated upon by the Schrödinger equation for a boson is either $\Psi(\varphi,t)=Ae^{im_r\varphi}e^{-i\omega t}$ or $\Psi^*(\varphi,t)=Ae^{-im_r\varphi}e^{i\omega t}$, the observable eigenvalue of the rotational energy equals $\frac{\hbar\omega}{2}$. The value of the transverse rotational energy of the binary photon is independent of the sense of the rotation and is equal to one-half of the total energy of the binary photon ($E_{total} = \hbar\omega$).

By definition, the product of $\Psi(\varphi, t)$ and $\Psi^*(\varphi, t)$ equals unity, which allows one to calculate the expectation values for the rotational energy $\langle E_{rotational}\rangle$ of the monochromatic binary photon using the Hamiltonian operator (Table 1) and the angular momentum of a monochromatic binary photon using the angular momentum operator (Table 1). The expectation value of the rotational energy is given by:

$$\langle E_{rotational}\rangle = \Psi^*(\varphi, t)\left(-\frac{\hbar^2 c^2 k^2}{2\hbar\omega}\right)\frac{\partial^2\Psi(\varphi,t)}{\partial\varphi^2} \quad (18)$$

$$\langle E_{rotational}\rangle =$$

$$e^{-im_r\varphi}e^{i\omega t}\left(-\frac{\hbar^2 c^2 k^2}{2\hbar\omega}\right)i^2 m_r^2\, e^{im_r\varphi}e^{-i\omega t} \quad (19)$$

Since $e^{-im_r\varphi}e^{i\omega t}e^{im_r\varphi}e^{-i\omega t} = 1$ and $m_r^2 = 1$, the expectation value[6] for the rotational energy of the monochromatic binary photon is:

$$\langle E_{rotational}\rangle = \frac{\hbar\omega}{2} \quad (20)$$

The expectation value for the rotational energy is equal to the eigenvalue for the rotational energy, which is one-half of the value of the total energy of the binary photon ($E_{total} = \hbar\omega$).

Using the angular momentum operator (Table 1), we can solve for the expectation value for the angular momentum $\langle L_z\rangle$ parallel or antiparallel to the propagation vector:

$$\langle L_z\rangle = \Psi^*(\varphi, t)\frac{\hbar}{i}\frac{\partial\Psi(\varphi,t)}{\partial\varphi} \quad (21)$$

$$\langle L_z\rangle = e^{-im_r\varphi}e^{i\omega t}\,\frac{\hbar}{i}\frac{\partial e^{im_r\varphi}e^{-i\omega t}}{\partial\varphi} \quad (22)$$

$$\langle L_z\rangle = e^{-im_r\varphi}e^{i\omega t}\left(\frac{im_r\hbar}{i}\right)e^{im_r\varphi}e^{-i\omega t} \quad (23)$$

Since $e^{-im_r\varphi}e^{i\omega t}e^{im_r\varphi}e^{-i\omega t} = 1$ and $m_r = \pm 1$, the expectation value for the angular momentum of the binary photon is:

$$\langle L_z\rangle = \frac{im_r\hbar}{i} = m_r\hbar = \pm\hbar \quad (24)$$

which is independent of the wave properties of the binary photon. Depending on the sign of m_r, $\langle L_z\rangle$ is equal to $\pm\hbar$. When $\langle L_z\rangle > 0$, the angular momentum vector is parallel to the propagation vector and $\langle L_z\rangle <$ 0, the angular momentum vector is antiparallel to the propagation vector.[7] Thus m_r is equivalent to the spin quantum number in standard quantum mechanics.

The individual positions of the positive mass semi-photon and the negative mass semi-photon that carry the rotational energy can be determined in principle using symmetry in the following manner: replace the position of the combined mass binary photon with the positive mass semi-photon; then the negative mass semi-photon can be located by finding the image of the positive mass semi-photon reflected by a plane mirror placed on the xz plane at the origin of the y-axis. When considering the two conjugate semi-photons separately, the time-averaged center of gravity of the binary photon remains constant and can be considered to be the rest frame of the binary photon.

The standing wave functions or eigenvectors for the rotational motion of monochromatic binary photons are degenerate in terms of their rotational energy but resolvable in terms of their angular momenta:

$$\psi_{m_r}(\varphi) = \begin{Bmatrix} A[\cos(\varphi) + i\sin(\varphi)] & m_r = +1 \\ A[\cos(\varphi) - i\sin(\varphi)] & m_r = -1 \end{Bmatrix} \quad (25).$$

Using the postulated wave function and the Schrödinger equation for a boson, the eigenvalue for the rotational energy of the binary photon around the axis of propagation only accounts for half of its total energy. Below I will show that the other half of the total energy is accounted for by the longitudinal translational and vibrational energy.

According to Maxwell's equations, the electromagnetic oscillations that make up light have no longitudinal component. However, this conclusion was based on the assumption that an electromagnetic wave in free space is neutral due to the absence of charge as opposed to the balance of charges [38]. Indeed, the longitudinal vibrational nature of light was not discounted by FitzGerald [40] and Röntgen [41].

[6] Here, the expectation value is for the monochromatic binary photon that is described by a wave with a single characteristic wave number (k); not by a set of numbers in a matrix nor by an infinite number of waves, where each wave with its characteristic wave number (k) has a different amplitude.

[7] According to the model of the binary photon, the angular momentum does not precess around the axis of propagation. The angular momentum has an eigenvalue along the axis of propagation only while the eigenvalues for the angular momentum vanish in the x-y plane. Note that for a given monochromatic binary photon, the angular momentum times the angular frequency of rotation is twice as large as the rotational kinetic energy and is equal to the negative of the transverse rotational potential energy.

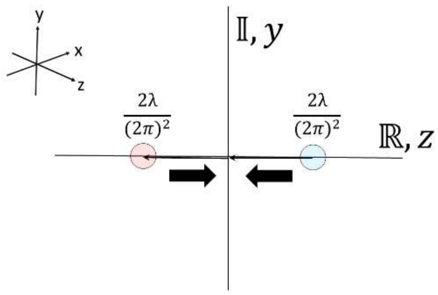

Fig. 4: The components of the longitudinal vibrational oscillation of a positive mass semi-photon (blue) and a negative mass semi-photon (red) parallel to the axis of propagation (z).

The longitudinal component of each semi-photon along the axis of propagation can be described in terms of simple harmonic motion. Relative to the time-averaged center of gravity, the positive mass semi-photon oscillates in the fore half of the binary photon while the negative mass semi-photon oscillates in the aft half (Fig. 4). We can convert a two-body problem into a one-body problem and simplify the construction of the wave function by combining the motion of two conjugate semi-photons that have masses and directions with opposite signs into the motion of one binary photon, with a mass that is twice the modulus of either semi-photon and moving with the same phase as the positive mass semi-photon. In the time that the combined mass binary photon completes one revolution in the transverse plane, the combined mass binary photon completes one complete longitudinal vibrational oscillation. I postulate that the wave function that describes the longitudinal oscillation of the binary photon is given by:

$$\Psi(z,t) = \frac{2A}{(2\pi)^2}(\cos^2 m_l kz - \omega t) \tag{26}$$

and in exponential terms that show two parts of the longitudinal vibrational wave function:

$$\Psi(z,t) =$$
$$\left(\frac{A}{8\pi^2} + A\frac{e^{2i\omega t - 2im_l kz}}{8\pi^2}\right) + \left(\frac{A}{8\pi^2} + A\frac{e^{-2i\omega t + 2im_l kz}}{8\pi^2}\right) \tag{27}$$

where A is the amplitude of the monochromatic wave function and equals unity, m_l is the longitudinal vibrational quantum number, k is the wave number and ω is the angular frequency of the monochromatic binary photon. The two parts of the exponential form of the wave function describe two waves that are a quadrature out-of-phase. Now we must check that the proposed wave function is a solution to the Schrödinger equation for a boson by separately substituting each part of Eqn. (27) into Eqn. (6):

$$\left(-\frac{\hbar^2 c^2}{2\hbar\omega}\right)\frac{\partial^2}{\partial z^2}\left(A\frac{e^{2i\omega t - 2im_l kz}}{8\pi^2}\right) =$$
$$\frac{i\hbar}{2}\frac{\partial}{\partial t}\left(\frac{A}{8\pi^2} + A\frac{e^{2i\omega t - 2im_l kz}}{8\pi^2}\right) \tag{28a}$$

$$\left(-\frac{\hbar^2 c^2}{2\hbar\omega}\right)\frac{\partial^2}{\partial z^2}\left(\frac{A}{8\pi^2} + A\frac{e^{-2i\omega t + 2i\quad {}_l kz}}{8\pi^2}\right) =$$
$$\frac{i\hbar}{2}\frac{\partial}{\partial t}\left(\frac{A}{8\pi^2} + A\frac{e^{-2i\omega t + 2im_l kz}}{8\pi^2}\right) \tag{28b}$$

Let $c^2 = \frac{\omega^2}{k^2}$, A =1, and differentiate

$$\left(-\frac{\hbar^2\omega^2}{2\hbar\omega k^2}\right)2^2 i^2 m_l^2 k^2\left(\frac{1}{8\pi^2} + \frac{e^{2i\omega t - 2im_l kz}}{8\pi^2}\right) =$$
$$\frac{i\hbar}{2}2i\omega\left(\frac{1}{8\pi^2} + \frac{e^{2i\omega t - 2im_l kz}}{8\pi^2}\right) \tag{29a}$$

$$\left(-\frac{\hbar^2\omega^2}{2\hbar\omega k^2}\right)2^2 i^2 m_l^2 k^2\left(\frac{1}{8\pi^2} + \frac{e^{-2i\omega t + 2im_l kz}}{8\pi^2}\right) =$$
$$-\frac{i\hbar}{2}2i\omega\left(\frac{1}{8\pi^2} + \frac{e^{-2i\omega t + 2im_l kz}}{8\pi^2}\right) \tag{29b}$$

After simplifying Eqn. (29), we get:

$$m_l^2 = \frac{i^2}{2} \tag{30a}$$

$$m_l^2 = \frac{1}{2} \tag{30b}$$

The first part of the longitudinal vibrational wave function (Eqn. 27) is a solution to the Schrödinger equation for a boson only if $m_l = \pm\frac{i}{\sqrt{2}}$ and the second part of the longitudinal vibrational wave function (Eqn. 27) is a solution to the Schrödinger equation for a boson only if $m_l = \pm\frac{1}{\sqrt{2}}$.

In order to find the energy eigenvalues for the longitudinal vibrational motion of the binary photon, we must separate the longitudinal vibrational wave function into its spatial and temporal parts that describe movement in absolute Euclidean space (z) and Newtonian time (t):

$$\Psi(z,t) = \psi(z)T(t) \tag{31}$$

To find the eigenvalues, substitute $\Psi(z,t) = \psi(z)T(t)$ into the Schrödinger equation for a boson:

$$\left(-\frac{\hbar^2 c^2}{2\hbar\omega}\right)\frac{\partial^2[\psi(z)T(t)]}{\partial z^2} = \frac{i\hbar}{2}\frac{\partial[\psi(z)T(t)]}{\partial t} \quad (32)$$

After treating the variables inside the partial derivatives that are not part of the partial derivative as constants, we get:

$$\left(-\frac{\hbar^2}{2m_{binary\ phot}}\right)T(t)\frac{\partial^2[\psi(Z)]}{\partial z^2} = \frac{i\hbar}{2}\,\psi(z)\frac{\partial[T(t)]}{\partial t} \quad (33)$$

Divide by $\psi(z)T(t)$:

$$\left(-\frac{\hbar^2}{2m_{binary\ photon}}\right)\frac{T(t)}{\psi(z)T(t)}\frac{\partial^2[\psi(z)]}{\partial z^2} = \frac{i\hbar}{2}\frac{\psi(z)}{\psi(z)T(t)}\frac{\partial[T(t)]}{\partial t} \quad (34)$$

Cancel like terms to get a fully separated equation where one term is only a function of Euclidean space (z) and the other term is only a function of Newtonian time (t).

$$\left(-\frac{\hbar^2}{2m_{binary\ photon}}\right)\frac{1}{\psi(z)}\frac{\partial^2[\psi(z)]}{\partial z^2} = \frac{i\hbar}{2}\frac{1}{T(t)}\frac{\partial[T(t)]}{\partial t} \quad (35)$$

After separating the variables, the partial differential equation becomes two ordinary differential equations that give the separation constant $E_{longitudinal}$, which is the eigenvalue for the total rotational energy of the binary photon.

$$E_{longitudinal} = \left(-\frac{\hbar^2}{2m_{binary\ photon}}\right)\frac{1}{\psi(z)}\frac{d^2[\psi(z)]}{dz^2} \quad (36a)$$

$$E_{longitudinal} = \frac{i\hbar}{2}\frac{1}{T(t)}\frac{d[T(t)]}{dt} \quad (36b)$$

Eqn. (36a) is the time-independent Schrodinger equation for the longitudinal vibrational energy of the binary photon. The solution gives the eigenvalues for the two parts of the longitudinal vibrational energy of the binary photon:

$$E_{longitudinal} = \left(-\frac{\hbar^2 c^2}{2\hbar\omega}\right)\frac{\partial^2}{\partial z^2}\left(\frac{1}{8\pi^2} + \frac{e^{2i\omega t - 2im_l kz}}{8\pi^2}\right) \quad (37a)$$

$$E_{longitudinal} = \left(-\frac{\hbar^2 c^2}{2\hbar\omega}\right)\frac{1}{\psi(z)}\frac{\partial^2}{\partial z^2}\left(\frac{1}{8\pi^2} + \frac{e^{-2i\omega t + 2im_l kz}}{8\pi^2}\right) \quad (37b)$$

Differentiate and simplify Eqns. (37a) and (37b):

$$E_{longitudinal} = -2^2 i^2 m_l^2 \frac{\hbar\omega}{2} \quad (38a)$$

$$E_{longitudinal} = -2^2 i^2 m_l^2 \frac{\hbar\omega}{2} \quad (38b)$$

Substitute $m_l^2 = \frac{i^2}{2}$ into Eqn. (38a) and $m_l^2 = \frac{1}{2}$ into Eqn. (38b) to get[8]:

$$E_{longitudinal} = -\hbar\omega \quad (39a)$$

$$E_{longitudinal} = +\hbar\omega \quad (39b)$$

Thus together, the two parts of the total longitudinal vibrational energy, which likely represent the potential and kinetic energies of the longitudinal oscillator, vanish in the binary photon. If the energy vanishes, why not consider the longitudinal vibration in the model of the binary photon to be superfluous and unnecessary? The longitudinal oscillation is required if one wants to prevent the two semi-photons from coming in contact with each other that would result in cycles of pair formation and annihilation occurring as a particle semi-photon made of matter and an antiparticle semi-photon made of antimatter collided in the transverse plane. The longitudinal vibration is also necessary to posit that the binary photon propagates as a result of a gravity-dependent Coulombic force and a velocity-dependent electromagnetic force acting on each semi-photon such that one semi-photon accelerates as the other semi-photon decelerates [34]. The gravitational force is the cause of the propagation and the induced electromagnetic force limits the velocity of the center of gravity of the binary photon to the speed of light [42].

Since the mass of the binary photon treated as a one-body problem is equal to $\frac{\hbar\omega}{c^2}$ and its velocity (v) is equal to c, its translational kinetic energy is given by:

$$E_{translational} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{\hbar\omega}{c^2}c^2 = \frac{\hbar\omega}{2} \quad (40)$$

The translational energy is considered to be positive when the binary photon interacts with matter, negative when the binary photon interacts with antimatter, and undetermined when the binary photon is in free space [42]. The linear momentum (p) of the binary photon treated as a one-body problem is given by:

[8] It is possible that this treatment could be applied to a simple harmonic oscillator where each quantum step would differ by $\hbar\omega$.

$$E_{translational} = \frac{p^2}{2m} \quad (41)$$

Since $E_{translational} = \frac{\hbar\omega}{2}$ and $m_{binary\ photon} = \frac{\hbar\omega}{c^2}$, the linear momentum ($p$) is given by:

$$p^2 = \frac{\hbar^2\omega^2}{c^2} \quad (42)$$

Since $c^2 = \frac{\omega^2}{k^2}$,

$$p = \hbar k \quad (43)$$

The linear momentum, like the translational energy is considered to be positive when the binary photon interacts with matter, negative when the binary photon interacts with antimatter, and undetermined when the binary photon is in free space.

The binary photon has two kinds of mechanical energies that are non-vanishing—transverse rotational and longitudinal translational; and two kinds of mechanical momenta—angular and linear. The translational energy is related to the linear momentum just as the rotational energy is related to the angular momentum. The total energy of the binary photon, which is equipartitioned into transverse rotational and longitudinal translational energy, is given by:

$$E_{total} = E_{rotational} + E_{translational} \quad (44a)$$

$$\hbar\omega = \left(\frac{\hbar\omega}{2}\right) + \left(\frac{\hbar\omega}{2}\right) \quad (44b)$$

Moreover, since $E_{total} = pc$ for a binary photon with a mass, then

$$E_{total} = hkc \quad (45)$$

If a binary photon is a propagating oscillating rotor and it is absorbed or emitted in its entirety, the quantum entity that absorbs or emits it must be able to exchange both the rotational and the longitudinal energy. Consequently, the absorber or emitter could be an oscillating rotor such as a Wilberforce spring [43] and not just a pure simple harmonic oscillator or a pure rotor (Fig. 5). Modeling the absorber or emitter as either a pure quantum harmonic oscillator or a pure quantum rotor results in a model that takes into consideration only one half of the energy exchange that occurs between matter and radiation and results in the postulate of a zero point energy equal to $\frac{\hbar\omega}{2}$— which is half a quantum [44-47][9]. By assuming that any real absorber or emitter equally exchanges transverse and longitudinal energy, the concept of zero point energy becomes superfluous.

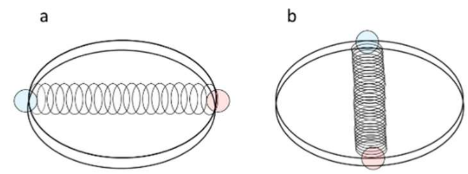

Fig. 5: Simplified model of an oscillating rotor or rotator with multiple degrees of freedom showing the oscillator in the stretched (a) and compressed (b) states.

De Broglie [31] postulated that the photon is a boson composed of two conjugate fermions. The two semi-photons that comprise the binary photon may be conjugate fermions (Appendix 2). The relationship between the angular momentum quantum numbers of a boson and a fermion support the idea that a boson is composed of two fermions [51]. If so, pair formation would be due to the binding of fermions into a photonic boson and annihilation would be due to the decomposition of bosonic photons into their constituent fermions [48-50]. The relationship between a boson and the conjugate fermions that comprise it make the relation between integral and half-integral quantization mechanically intelligible.

In cylindrical coordinates, the complete mechanical wave function of a propagating monochromatic binary photon is:

$$\Psi(\varphi, z, t) = \left[\frac{Ae^{i(m_r\varphi-\omega t)}}{ct + \frac{2A}{(2\pi)^2}\left(\frac{(\cos^2 m_l kz-\omega t)}{2\pi^2}\right)}\right] \quad (46)$$

The quantum number (m_r) for the binary photon is equivalent to the rotational quantum number (m [46]; J [47]) in standard quantum mechanics, and the quantum number (m_l) for the binary photon is equivalent to the vibrational quantum number (n [46]; v [47]) in standard quantum mechanics.

The standard model of the photon as a mathematical point is inconsistent with the entropy of the photon being 3.60k [52] based on blackbody

[9] The zero point energy vanished when Planck assumed that the absorption and emission of radiation from an oscillator was discontinuous and appeared when he assumed that the absorption of radiation was continuous and the emission was discontinuous [3,44]. The zero point energy disappears as long as the absorption and emission of a binary photon changes the energy of the oscillator by $\hbar\omega$, which accounts for the equipartitioned transverse ($\frac{\hbar\omega}{2}$) and longitudinal energy ($\frac{\hbar\omega}{2}$).

radiation theory. The internal motions posited for the binary photon are more consistent with the observed entropy if the translational energy contributes $\frac{3}{2}$k, the transverse rotational energy contributes $\frac{2}{2}$k, and the longitudinal vibrational energy contributes $\frac{2}{2}$k for a total entropy of about 3.5k. The space in which the proposed motions take place is also consistent with the size of a photon given independently by Silberstein [24, 25], Ornstein and Burger [26], Marx [27], Lehnert [28] and Wayne [29]. The extended photon was described by Silberstein [24,25,52,54] as light darts, Oseen [55] as an "*Einsteinschen Nadelstichstrahlung*," Weber [56] as "*Nadelstrahlung*," Debye [57,58] as "*Nadelstrahlung*," Pauli [59] as "*needle radiation*" and Thomson [60,61] as a discrete, "*cylindrical pencil*."

The development of a theory of the photon as a composite particle consisting of a neutrino and an antineutrino came to a halt when Pryce [62] and Case [63] determined that such a theory could not simultaneously satisfy both quantum mechanical commutation rules and the Lorentz invariance of the theory of relativity. The model of the binary photon presented here provides a firm foundation for rethinking the rejection of *all* models of binary photons.

The model of the binary photon also gives us reason to rethink the meaning of the wave function. According to standard quantum mechanics, particles do not possess definite positions and do not follow definite trajectories in Euclidean space and Newtonian time during the periods between measurements. According to standard quantum mechanics, the wave function does not describe a trajectory. The product of the wave function and its complex conjugate gives only the probability of finding a particle at a given position or with a given velocity. Before measurement, the particle does not follow a knowable trajectory in Euclidean space and Newtonian time. By contrast, the wave functions given here and their derivatives are a prescription for knowing the positions, velocities, and momenta of the semi-photons that make up the binary photon at a given time if one knows the initial positions in Euclidean space and Newtonian time.

Solving the Schrödinger equation, where the spatial derivative is second order and the temporal derivative is first order, requires the use of imaginary numbers whose meanings are celebrated as being unintuitive and difficult to picture. The complex wave function presented in Eqns. (7a) and (7b) exist in the transverse plane perpendicular to the axis of propagation in ordinary space.

I have discussed the importance of the mechanical properties of light in preventing particles with a charge and/or a magnetic moment from exceeding the speed of light in Euclidean space and Newtonian time [29]. Here I have developed and used the Schrödinger equation for a boson to extract the mechanical properties of light based on the mass and parity of the semi-photons. In a companion paper [38], I apply wave mechanics to the complete binary photon composed of conjugate semi-photons with mass, parity and charge.

The model of the binary photon has previously been put to a test by being able to describe and explain the observed magnitude of the gravitational deflection of starlight—the *experimentum crucis* in favor of the general theory of relativity [34,64]. The model of the binary photon, which assumes that the binary photon rotates as it translates through absolute space and time, gives the same prediction as Einstein's general theory of relativity that posits that mass warps spacetime through which a geometrical point-like photon propagates. This means that the interpretation of the *experimentum crucis* depends on the model of the photon. If the photon is a mathematical point whose energy cannot be partitioned into translational and rotational energy, then space and time must be relative and interdependent and exist as space-time. However, if the photon is a composite particle, whose total energy is equipartitioned between its transverse and longitudinal components, then space must be Euclidean and time must be Newtonian.

3. Limitations of the Theory

Quantum theory, at its essence, recognizes the discrete nature of elementary processes and proposes that the continuous nature of natural macroscopic processes is a result of the summation of many elementary and discrete processes. On the other hand, the philosophical impact of quantum mechanics has led to the idea that "*reality is what you choose it to be*" [65].

I have returned to the essence of quantum theory as described above and tried to model the elementary quantum of light, which I assume to be the binary photon, using small integers and a minimum of free parameters. This seemed to work well for the rotational energy. However, the longitudinal vibrational wave function has more free parameters than I would like. Initially, I guessed that the coefficient that describes the amplitude of the wave function would be $\frac{1}{4}$ and the wave form would be described by a cosine function. This seemed reasonable since it would give a mechanical length of the binary photon that would be equal to the wavelength of light. However, with this value, I later found that the transverse electric field produced by the binary photon did not have the form of a sinusoidal wave. Since the ability to generate a sinusoidal transverse electrical field is a *sine qua non* for a correct model of the photon, I had to search for a coefficient and a functional form that would produce a sinusoidal transverse electrical field based on small integers. If the longitudinal mechanical length vanished, then, as mentioned above, the semi-photons would collide with each other as they rotate in the transverse plane. The smallest non vanishing coefficient that gives a sinusoidal transverse electrical waveform is $\frac{2}{(2\pi)^2}$.

Another limitation of the theory is that I assume that the charge of each semi-photon is $\pm 1.602 \times 10^{-19}$ C. This should be considered a free parameter, whose justification is weak.

The next limitation is that the longitudinal electric field of the binary photon is not extinguished when two binary photons that are out-of-phase by a half wavelength interfere [66]. Again, total destructive interference is a *sine qua non* for a correct model of the photon. Consequently I had to assume that two classes of photons were simultaneously emitted from matter. One class would be emitted from orbital transitions that emitted binary photons with angular momentum of $+\hbar$ and the other class would be emitted from orbital transitions that emitted binary photons with angular momentum of $-\hbar$. This only requires that the atoms emitting light are randomly oriented, which seems reasonable.

4. Conclusions

Here I have shown that the three-dimensional motions of the semi-photons that make up a binary photon can be described by continuous wave functions. The transverse wave function describes the rotational motion of the semi-photons in real space perpendicular to the axis of propagation and the longitudinal vibrational wave function describes the movement of the semi-photons in real space along the axis of propagation. The wave functions give *Anschaulichkeit* or picturability to the movements of the semi-photons within the volume of a monochromatic binary photon. Such picturability, which allows for classical trajectories and was a *sine qua non* of a complete physical theory according to Einstein [67], was rejected by Born, Heisenberg, and Bohr as a foundational principle of the "*Kopenhagener Geist der Quantentheorie* [68]" or the Copenhagen interpretation of quantum mechanics [69-76]. The editor of *Nature* wrote in an introduction to one of Bohr's papers, "*It must be confessed that the new quantum mechanics is far from satisfying the requirements of the layman who seeks to clothe his conceptions in figurative language. Indeed, its originators probably hold that such symbolic representation is inherently impossible. It is earnestly to be hoped that this is not their last word on the subject, and that they may yet be successful in expressing the quantum postulate in picturesque form* [77, 78]."

Einstein wrote to Born, "*Of this I am firmly convinced that we shall eventually land at a theory in which the things that are linked by laws are not probabilities but imaged facts, as was taken for granted until lately* [79,80]." Born [81] called this attitude, "*naïve realism*" believing that "*everything is subjective. Everything without exception*." Einstein has been called "*the last classical physicist* [82]," and when Einstein lost the battle with Born over the principle of objective truth, the existence of any other objective truth besides quantum chance was rejected not only by physicists but also by many in the postmodern world [82,83]. Heisenberg [68] wrote, "*words can only describe things of which we can form mental pictures, and this ability, too, is a result of daily experience. Fortunately, mathematics is not subject to this limitation, and it has been possible to invent a mathematical scheme—the quantum theory—which seems entirely adequate for the treatment of atomic processes; for visualization, however, we must content ourselves with two incomplete analogies—the wave picture and the corpuscular picture The simultaneous applicability of both pictures is thus a natural criterion to determine how far each analogy may be 'pushed' and forms an obvious starting point for the critique of the concepts which have entered atomic theories….*" In the microworld of the binary photon as presented here, such an objective view that encompasses *anschaulichkeit*, picturability, visualizability and images of classical trajectories are actually possible in principle.

5. Appendix 1

Imaginary numbers have a fascinating history. The first occurrence of a square root of a negative number is found in the *Stereometrica* of Hero of Alexandria where he calculated the volume of a frustum of a pyramid [84]. The square roots of negative numbers were called sophisticated numbers by Girolamo Cardano because they didn't seem to have any physical meaning [85]. In 1637 in his *Geometrie,* René Descartes [86] described square roots of negative numbers as imaginary numbers. At a time when any negative number was looked on with suspicion, John Wallis [87] developed the number line with positive numbers to the right of zero and negative numbers to the left and claimed, "*when it comes to a Physical Application, it* [a negative quantity] *denotes as Real a Quantity as if the Sign were +; but to be interpreted in a contrary sense*." From this starting point, Wallis began to look at the physical meaning square root of negative numbers using geometrical constructions.

Leonhard Euler [88] described complex numbers that are composed of a real term and an imaginary term containing $\sqrt{-1}$, which he designated by the symbol i. In 1799, Caspar Wessel [89], a surveyor who dealt with practical problems involved with map making, developed a graphical representation for real and imaginary numbers. Wessel considered complex numbers to form an ordered pair ($a + ib$) similar to latitude and longitude, that consisted of a real (a) part and an imaginary (ib) part that could be represented by a plane. In Cartesian coordinates, the real part of the complex number is the abscissa plotted with respect to the horizontal axis and the imaginary part is the ordinate plotted with respect to the vertical axis. To Wessel, the complex number could also be represented in polar coordinates as the length ($\sqrt{a^2+b^2}$) of a radial vector from the origin and the angle ($\theta = \tan^{-1}\left(\frac{b}{a}\right)$) measured by an anticlockwise rotation from the horizontal axis. To Wessel, the vertical imaginary axis is perpendicular to the horizontal real axis, $a + ib = \sqrt{a^2+b^2}[\cos\theta + i\sin\theta]$, and multiplying by i results in a 90° anticlockwise rotation. The geometrical representation of complex numbers was lost until 1806 when Jean-Robert Argand [90], a bookseller, independently rediscovered the geometrical representation of complex numbers. The geometrical representation of complex numbers only became widely known after Gauss' [91] independent rediscovery in 1831 and the complex plane is often referred to as the Argand or Gaussian plane. Complex numbers have many uses in physics and engineering [85] where they relate two distinct parts to a whole. Since the introduction of Einstein's theories of relativity, i has achieved fame and an enigmatic reputation in representing imaginary time [10] in the space-time metric

$$s^2 = x^2 + y^2 + z^2 + i^2c^2t^2 \tag{A1}$$

in which the space-time interval (s) is invariant. On the other hand, I have shown that the observations upon which special and general relativity were founded can be explained in terms of absolute Euclidean Space and Newtonian time [29,51,64] without invoking relative or imaginary time [92,93].

Appendix 2

According to de Broglie [31], the photon is a boson composed of two conjugate fermions. I have characterized the binary photon in terms of two complex wave functions that describe separately the transverse energy and the longitudinal energy relative to the time-averaged center of gravity of the binary photon. Since I postulate that the longitudinal vibration of the binary photon results from the position-dependent gravitational and the velocity-dependent electromagnetic forces exerted on each conjugate particles by the other [34], I assume that the longitudinal vibrational energy is an interaction energy that appears only when the conjugate fermions come together and vanishes when the conjugate fermions are separated. Moreover, a fermion, in the absence of a conjugate fermion to screen its charge, cannot travel at nor exceed the speed of light [29]. The energy released in the binding of the two semi-photons is equal to the translational energy of the binary photons so that the total energy of the binary photon is given by the sum of the rotational and translational energies.

Without the conjugate particle, and in the absence of an accelerating field, a fermion (with half the mass as the boson it makes up) has intrinsic rotational energy or spin. Since the total energy of the fermion is not equipartitioned between transverse rotational and longitudinal energy, to compensate, the radius of rotation of the isolated fermion must shrink to one-half of the value it had when bound in the boson and its energy was equipartitioned between transverse rotational and longitudinal energy. That is, the total mechanical internal energy of a boson is equipartitioned between the transverse rotational and longitudinal energy while the total intrinsic mechanical internal energy of a fermion is exclusively rotational.

I assume that the fermions or semi-photons, like the boson described above, have extension in ordinary space. Consequently, the wave function for an isolated semi-photon or a fermion rotating with an angular frequency (ω) is given by:

$$\Psi(\varphi, t) = Ae^{im_f\varphi}e^{-i\omega t} \tag{A2}$$

where m_f is the rotational quantum number of a fermion. When using the Schrödinger equation for a fermion, the modulus of the mass of the fermion that makes up a boson of mass $\frac{\hbar\omega}{c^2}$ is $\frac{\hbar\omega}{2c^2}$. With the mass reduced to ($\frac{\hbar\omega}{2c^2}$) and the reciprocal of the radius reduced to ($r = \frac{1}{2k}$), the moment of inertia of an isolated semiphoton or fermion would be:

$$I = m_{semiphoton}r^2 = \frac{\hbar\omega}{2c^2(2k)^2} = \frac{\hbar\omega}{8c^2k^2} \tag{A3}$$

The Schrödinger equation for a rotating fermion ($n = 1$) becomes:

$$\left(-\frac{\hbar^2}{2mr^2}\right)\frac{\partial^2 A e^{im_f\varphi} e^{-i\omega t}}{\partial\varphi^2} = i\hbar\frac{\partial A e^{im_f\varphi} e^{-i\omega t}}{\partial t} \quad \text{(A4)}$$

Using the reduced radius,
let $m_{semiphoton} r^2 = \frac{8c^2k^2}{\hbar\omega}$:

$$\left(-\frac{8\hbar^2c^2k^2}{2\hbar\omega}\right)\frac{\partial^2 A e^{im_f\varphi} e^{-i\omega}}{\partial\varphi^2} = i\hbar\frac{\partial A e^{im_f\varphi} e^{-i\omega t}}{\partial t} \quad \text{(A5)}$$

After simplifying and differentiating, we get:

$$\left(-\frac{4\hbar^2c^2k^2}{\hbar\omega}\right) i^2 m_f^2 A e^{im_f\varphi} e^{-i\omega t} = -i^2\hbar\omega A e^{im_f\varphi} e^{-i\omega t} \quad \text{(A6)}$$

After simplifying, we get:

$$m_f^2 = \frac{1}{4} \quad \text{(A7)}$$

Consequently, $m_f = \pm\frac{1}{2}$. Thus Eqn. (A2) is only a solution to the Schrödinger equation for fermions if $m_f = \pm\frac{1}{2}$. This is equivalent to the spin quantum number for a fermion in standard quantum mechanics. When $m_f = +\frac{1}{2}$, the intrinsic spin of a fermion with positive mass is anticlockwise and when $m_f = -\frac{1}{2}$, the intrinsic spin of a fermion with positive mass is clockwise. The expectation value of the angular momentum $\langle L_z\rangle$ of the isolated semiphoton or fermion is given by:

$$\langle L_z\rangle = \psi^*(\varphi,t)\frac{\hbar}{i}\frac{\partial e^{im_f\varphi} e^{-i\omega t}}{\partial\varphi} = \hbar m_f = \pm\frac{1}{2}\hbar \quad \text{(A8)}$$

which is equal to the intrinsic angular momentum of a spin ½ particle like a fermion according to standard quantum mechanics. When $m_f = +\frac{1}{2}$, the angular momentum of a translating fermion is parallel to the axis of propagation and when $m_f = -\frac{1}{2}$, the angular momentum of a translating fermion is antiparallel to the axis of propagation.

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